Engaging Students in Mathematical Modelling and Problem Posing Activities

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Abstract
In this paper we want discuss the results of some teaching experiences based on the classroom activities that are more easily related to the experiential world of the student and consistent with a sense-making disposition. They make extensive use of cultural artifacts that could prove to be useful instruments in creating a new tension between school mathematics and real world with its incorporated mathematics. The classroom activities are also based on the use of a variety of complementary, integrated, and interactive teaching methods, and on the introduction of new socio-mathematical norms, in an attempt to create a substantially modified teaching/learning environment.

The idea is not only to motivate students with everyday-life contexts but also to look for contexts that are experientially real for the students and can be used as starting points also for progressive mathematization, in order to favour a mindful approach toward realistic mathematical modeling and a problem-posing attitude as encouraged by recent Italian document for new curricula.

Keywords: Mathematical Modelling, Problem Posing, Teaching Experiments

1. Introduction

In the Italian National Curricular Recommendations for the primary school and first cycle of instruction we read: “A characteristic of practical mathematics is the solution of problems, which must be viewed as authentic, meaningful questions, often linked to daily life, and not mere repetitive exercises or questions to which the children answer simply by remembering a procedure or a rule”.

We find similar recommendations also in the documents of the European Community concerning competences, particularly mathematical competence: “Competences are defined here as a combination of knowledge, skills and attitudes appropriate to the context. Key competences are those which all individuals need for personal fulfillment and development, active citizenship, social inclusion and employment. ... Mathematical competence is the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations. Building on a sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge.”

The connection between everyday mathematics and in-of-school mathematics is not easy to make because the two contexts differ significantly. Just as mathematical practice differs in and out of school (Lave, 1988; Nunes, 1993) so does mathematics learning (Resnick, 1987). Masingila, Davidenko, and Prus-Wisniowska (1996) outlined three key differences between in- and out-of-school practices (goals of the activity, conceptual understanding, and flexibility in dealing with constraints).

In out-of-school mathematical practice in particular, people may generalize procedures within one context, but may not be able to generalize to another, since problems tend to be context specific. Generalization, an important goal in school mathematics, is not usually a goal in out-of-school mathematics.

Furthermore, as regard the relationship between mathematics in the workplace and scholastic mathematics, in the workplace mathematics makes sophisticated use of elementary mathematics rather than, as in the classroom, elementary use of sophisticated mathematics. Work related mathematics is rich in data, interspersed with conjecture, dependent on technology, and tied to useful applications.

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1 Ministry Decree of July 31, 2007- Enclosure on National Curricular Recommendations for the primary school and first cycle of instruction.
Work contexts often require multistep solutions to open-ended problems, a high degree of accuracy, and proper regard for required tolerances. None of these features is found in typical classroom exercises (Steen, 2003).

In normal teaching practice establishing connections between classroom mathematics activities and real world still regards mainly word problems. But besides representing the interplay between formal mathematics and reality, word problems are often the only means of providing students with a basic sense experience in mathematization and mathematical modeling (Bonotto, 2005).

Many researchers have documented that the practice of word problems solving in school mathematics actually promotes in students a suspension of sense-making (Schoenfeld, 1991), and the exclusion of realistic considerations. Primary and secondary school students tend to exclude relevant and plausible familiar aspects of reality from their observation and reasoning (for a comprehensive overview of these studies see Verschaffel, Greer, & De Corte, 2000).

Several studies point to two reasons for this lack of use of everyday-life knowledge: textual factors relating to the stereotypical nature of the most frequently used textbook problems and presentational or contextual factors associated with practices, environments and expectations related to the classroom culture of mathematical problem solving (see e.g. Gravemeijer, 1997; Palm, 2008). For example teaching practice in Italian schools is still based on a type of didactic contract that has the following foundations:

- **every problem has one and only one solution.** The students learn from the earliest grades that every problem has a single right answer that is obtained by the simple mechanical application of the four operations (addition, subtraction, multiplication, division). Whole generations of authors of schoolbooks have written texts full of word problems that can always be solved because that is the usual practice. The students, in turn, in their desire to satisfy their teachers’ expectations, learn to look for the right answer, the one that will lead to the result shown at the end of the exercise;

- **the numbers are all indispensable.** Another assumption shared by the children is that the numbers they encounter in the problems are all necessary (and sufficient) to find the answer. For the majority of children word problems are a formal linguistic structure, characterized by a text in which there are numbers that must be used to perform certain operations;

- **the form is more important than the correctness.** In the primary grades, the child, even if he can solve a simple problem quickly and mentally, is forced to make a series of representations of procedures that, of course, in this case lose their “helpful” function and become a useless ritual that inhibits any mathematical creativity and the search for alternative methods.

Furthermore, it has been noted that the use of stereotyped problems and the accompanying classroom climate relate to teachers’ beliefs about the goals of mathematics education. In the researches of Verschaffel, De Corte, and Borghart (1997) and Bonotto & Wilczewski (2007), student-teachers were asked to judge realistic and non-realistic answers to problematic word problems. The research shows that the student-teachers’ overall evaluation of the stereotyped, non-realistic answers to these items was considerably more positive than for the realistic answers grounded in context-based considerations. The teachers seem to believe that the activation of realistic context-based considerations should not be stimulated but rather discouraged in elementary-school mathematics.

Finally another reason for the abstention from using realistic considerations is that the practice of word problem solving is relegated to classroom activities, having meaning and location, in terms of time and space, only within the school; rarely will students encounter these activities in this form outside of school (Bonotto, 2005).

This indicates a difference in views on the function of word problems in mathematics education. The researchers relate word problems to problem solving and applications. The student-teachers (and probably teachers in general) see word problems as nothing more, and nothing less, than exercises in the four basic operations which also have a justification and suitable place within the teaching of mathematics, though certainly not that of favoring realistic mathematical modeling (Blum and Niss, 1991).

If we wish to establish situations of realistic mathematical modeling we have to: i) to change the type of activity aimed at creating interplay between real world and mathematics with more realistic
and less stereotyped problem situations; ii) to change students’ conceptions of, beliefs about and attitudes towards mathematics; this means changing teachers’ conceptions, beliefs and attitudes as well; iii) to change classroom culture by establishing also new classroom socio-mathematical norms.

In this paper, based on results obtained in several teaching experiments, we want to discuss how these changes can be brought about at primary school level through classroom activities that are more easily related to the experiential world of the student and consistent with a sense-making disposition. They make extensive use of cultural artifacts that could prove to be useful instruments in creating a new tension between school mathematics and real world with its incorporated mathematics. The classroom activities are also based on the use of a variety of complementary, integrated, and interactive teaching methods, and on the introduction of new socio-mathematical norms (Yackel and Cobb, 1996), in an attempt to create a substantially modified teaching/learning environment.

The idea is not only to motivate students with everyday-life contexts but also to look for contexts that are experientially real for the students and can be used as starting points also for progressive mathematization (Gravemeijer, 1999), in order to favour a mindful approach toward realistic mathematical modeling and a problem-posing attitude, as encouraged also by recent Italian document for new curricula. There are obvious relations of modeling with problem posing and problem solving, “representative of a new thinking in Mathematics Education” (D’Ambrosio, 2009).

2. About Mathematical Modeling

Mathematical modeling is an important component of professional training, which is very much alike in all areas, particularly in mathematics education. The incorporation of mathematical modelling in mathematics education leads, indeed, to creating a learning environment. This requires a new approach to the objectives teaching mathematics. The learning environment thus created is a form of “open classroom” practice (D’Ambrosio, 2009).

Various views of the modeling process co-exist within educational circles. These have to do both with perceptions of the modeling process, and constraints and opportunities perceived to exist within particular educational setting.

The term mathematical modeling is not only used to refer to a process whereby a situation has to be problematized and understood, translated into mathematics, worked out mathematically, translated back into the original (real-world) situation, evaluated and communicated. Besides this type of modelling, which requires that the student has already at his disposal at least some mathematical models and tools to mathematize, there is another kind of modelling, wherein model-eliciting activities are used as a vehicle for the development (rather than the application) of mathematical concepts (Greer, Verschaffel & Mukhopadhyay, 2007).

This second type of modeling is called ‘emergent modeling’ (Gravemeijer, 2007), and its focus is on long-term learning processes, in which a model develops from an informal, situated model (“a model of”), into a generalizable mathematical structure (“a model for”).

“These emergent models are seen as originating from activity in, and reasoning about situations. From this perspective, the process of constructing models is one of progressive reorganizing situations. The model and the situation being modeled co-evolve and are mutually constituted in the course of modelling activity” (Gravemeijer, 2007).

Although it is very difficult, if not impossible, to make a sharp distinction between the two aspects of mathematical modelling, it is clear that they are associated with different phases in the teaching/learning process and with different kinds of instructional activities (Greer et al., 2007). However, in this contribution the focus will be more addressed to the second aspect of mathematical modelling.

An early introduction in schools of fundamental ideas about modelling is not only possible but also indeed desirable even at the primary school level (see also Usiskin, 2007, for some suggestions of ways to reconsider for example arithmetic operations as mathematical models).

“It might be appropriate to introduce the modelling perspective much earlier in the child’s education … in order to prevent – rather than remedy – routine behaviour and to continue this preventive effort throughout the mathematics curriculum” (De Bock, Van Dooren, Janssens, 2007).

Further we will argue for modelling can be seen as a means of recognizing the potential of mathematics as a critical tool to interpret and understand reality, the communities children live in, or
society in general. Teaching students to interpret critically the reality they live in and to understand its codes and messages so as not to be excluded or misled, should be an important goal for compulsory education (Bonotto, 2005).

To implement an early introduction in elementary schools of fundamental ideas about realistic mathematical modeling, and for laying the foundations of a mathematization disposition, we deem that the type of activity used to create an interplay between mathematics classroom activities and everyday-life experience must be replaced with more realistic and less stereotyped problem situations. These should be more closely related to children’s experiential world and meaningful. I deem that an extensive use of suitable artifacts could be a useful instrument in creating a new link between school mathematics and everyday-life with its incorporated mathematics, by bringing students’ everyday-life experiences and reasoning into play.

Finally, I believe that certain cultural artifacts lend themselves naturally to helping students with problem posing activities.

3. About Problem Posing

It is well recognized that problem posing is an important component of the mathematical curriculum and, indeed, lies at the heart of mathematical activity (English, 1998). Not surprisingly, reports such as those produced by the National Council of Teachers of Mathematics have called for an increased emphasis on problem-posing activities in the mathematics classroom.

Problem posing and problem solving are closely related. As Silver (1994) suggested, problem posing could occur prior to problem solving when problems were being generated from a particular situation or after solving a problem when experiences from the problem-solving context are modified or applied to new situations. In addition, problem posing could occur during problem solving when the individual intentionally changes goals while in the process of solving the problem.

Given the importance of problem-posing activities in school mathematics, some researchers started to investigate various aspects of problem-posing processes (Silver, 1994; Silver and Cai, 1996; Leung, 1996; English, 1998 and 2003; Christou et al., 2005). Several studies have reported approaches to incorporate problem posing in instruction. Some studies provided evidence that problem posing has a positive influence on students’ ability to solve word problems and provided a chance to gain insight into students’ understanding of mathematical concepts and processes. It was found that students’ experience with problem posing enhances their perception of the subject, provides good opportunities for children to link their own interests with all aspects of their mathematics education, and can prepare students’ to be intelligent users of mathematics in their everyday lives.

Despite its significance in the curriculum, problem posing has not received the attention it warrants from the mathematics education community. Little is known about the nature of the underlying thinking processes that constitute problem posing, and the schemes through which students’ mathematical problem posing can be analyzed and assessed (Christou et al., 2005). We know comparatively little about children’s ability to create their own problems in both numerical and non-numerical contexts or about the extent to which these abilities are linked to competence in problem solving. Research on these issues is particularly warranted, given the well-documented evidence that young children’s creativity and open-mindedness in generating and solving problems dissipate as they progress toward the higher school grades (English, 1998).

Problem posing has been defined by researchers from different perspectives (see Silver and Cai, 1996). In this contribution we consider mathematical problem posing as the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems. It, therefore, becomes an opportunity for interpretation and analysis of reality in different ways: i) they have to distinguish significant data from irrelevant data; ii) they must discover the relations between the facts; iii) they must decide whether the information in their possession is sufficient to solve the problem; and iv) they must investigate if numerical data involved is numerically and/or contextually coherent. These activities, quite absent from today’s school context, are typical of the modelling process and can help students to prepare to cope with natural situations they will have to face out of school (Bonotto, 2009).

According to English (1998) “we need to broaden the types of problem experiences we present to children … and, in so doing, help
children “connect” with school mathematics by encouraging everyday problem posing (Resnick et al., 1991). We can capitalize on the informal activities situated in children’s daily lives and get children in the habit of recognizing mathematical situations wherever they might be”.

4. About Artifacts

For some years now our research has been concerned with the following problematic: i) how can we benefit from the numerical culture children acquired outside the school while simultaneously avoiding the strengths and limitations that are typical of the usual everyday mathematics, and ii) how can we design better opportunities for children to develop new understandings about underlying mathematical concepts and structures and their potential generalizability, in a way that preserves the focus on meaning found in everyday situations.

In our approach in- and out-of-school mathematics, even with their specific differences, in terms both of practices and learning processes, are not seen as two disjoint and independent entities. Although the specificity of both contexts is recognized, we deem that those conditions that often make extra-school learning more effective (see e.g. Nunes T., Schielmann D. & Carraher, 1994) can and must be re-created, at least partially, in classroom activities. Indeed while some differences between the two contexts may be inherent, many differences can be narrowed by creating classroom situations that promote learning processes closer to those arising from out-of-school mathematics practices (Bonotto, 2005).

That can be implemented in a classroom by encouraging the children to analyze some mathematical ‘facts’, which are embedded in opportune ‘cultural’ or ‘social’ artifacts; these mathematical ‘facts’ can be seen as concrete extensions [in the logical sense] of the mathematical concept, which have instead intentional nature [in the logical sense].

The artifacts introduced into classroom activities (for example labels and supermarket receipts, menu of restaurants and pizzerias, advertising leaflets containing discount coupons for supermarkets and stores, the weather forecast from a newspaper, a weekly TV guide, an informational booklet issued by “Poste Italiane”, and so on) are materials, real or reproduced, which children typically meet in real-life situations and then are relevant and meaningful. In this way we offer the opportunity of making connections between the mathematics incorporated in real-life situations and school mathematics, which although closely related, are governed by different laws and principles. In this way we present mathematics as a means of interpreting and understanding reality and increase the opportunities of observing mathematics outside the school context. The usefulness and pervasive character of mathematics are merely two of its many facets and cannot by themselves capture its very special character, relevance, and cultural value. Nonetheless, these two elements can be usefully exploited from a teaching point of view because they can change students’ common behavior and attitude.

The use of artifacts in our teaching experiments has been articulated in various stages, with different educational and content objectives.

First, the dual nature of the artifacts, that is belonging to the world of everyday life and to the world of symbols, to use Freudenthal’s expression, allows movement from situations of normal use to the underlying mathematical structure and vice versa, in agreement with ‘horizontal mathematization’ (Treffers, 1987).

But these artifacts may also become real “mathematizing tools” with some modification, e.g. removing some data that are present in the artifacts (see e.g. Bonotto, 2005); in this way we can create new mathematical goals and provide students with a basic experience in mathematical modelling. In this new role, the cultural artifact can be used as motivating stepping-stones to launch new mathematical knowledge, through the particular learning processes that Freudenthal (1991) defines ‘prospective learning’ or ‘anticipatory learning’; it thus also becomes mathematizing tool that preserve the focus on meaning found in everyday situations.

We think that the prospective learning is better enhanced by a “rich context” as outlined by Freudenthal, that is a context which does not only serve as the application area but also as source for learning mathematics. The artifacts and classroom activities we introduced fall under this type of context.

These experiences have also favored the type of learning “retrospective” that occurs when old
notions are recalled in order to be considered at a higher level and within a broader context, a process typical of adult mathematicians. This different use of the artifacts made it possible to carry out, compatibly with the grade level, also vertical mathematization, from concepts to concepts, which may be described as the process of reorganization within the mathematical system itself. This occurred when symbols, i.e. embedded mathematical facts, became objects to be put in relationship, modified, manipulated, and reflected upon by the children through property noticing, conjecturing, and problem solving.

The following are some examples of artifacts introduced into classroom activities with briefly specified the content learning goal:

- some supermarket bills to introduce some aspects of multiplicative structure of decimal numbers (Bonotto, 2005),
- some menus of restaurants and pizzerias to enhance the understanding of decimal numbers (Bonotto, 2006), in particular of what Hiebert, 1985, calls site 1 (“symbols and their referents”), and site 3 (“solutions and their reasonableness in light of other knowledge”),
- a ruler to enhance the construction of a comprehensive numerical structure, which integrates in a whole both the natural and the decimal number systems (Bonotto, 2001),
- a cover of a ring binder to introduce the concept of surface area (Bonotto, 2003a),
- a weekly TV guide to develop the concept of equivalence between time intervals expressed in different ways (Bonotto, 2003b),
- advertising leaflets containing discount coupons for supermarkets and stores to develop the concept of percentage (Bonotto & Baroni, 2008),
- an informational booklet issued by “Poste Italiane” to estimate and discover area and length dimensions of some envelopes (Bonotto & Ceroni, 2003),
- the maps of cities to introduce and practice with the system of Cartesian coordinates (Bonotto & Baroni, 2010).

Furthermore the use of suitable artifacts allows the teacher to propose many questions, remarks, and culturally and scientifically interesting inquiries. The activities and connections that can be made depend, of course, on the students’ scholastic level. These artifacts contain different codes, percentages, numerical expressions, and different quantities with their related units of measure, and hence are connected with other mathematical concepts and also other disciplines (chemistry, biology, geography, astronomy, etc.).

It could be said that the artifacts are related to mathematics (and other disciplines) as long as one is able to form these relationships.

However several educators (e.g. Vygotsky, 1978, Schliemann, 2002, Saxe, 2002) have noted that is not the artifact (or tool) in isolation that offers support to the teacher – rather the student use of the tool and the meanings they have developed as a result of the activity.

Finally by asking children i) to select other artifacts from their everyday life, ii) to identify the embedded mathematical facts, iii) to look for analogies and differences (e.g. different number representations), iv) to generate problems (e.g. discover relationships between quantities) the children should be encouraged to recognize a great variety of situations as mathematical situations, or more precisely mathematizable situations. A “re-mathematization” process is thereby favored, wherein students are invited to unpack from artifacts the mathematics that has been “hidden” in them, in contrast with the demathematization process in which the need to understand mathematics that becomes embodies in artifacts disappears (see Gellert & Jablonka, 2007). In this way we can multiply the occasions when students encounter mathematics outside of the school context, “everydaying” the mathematics. So we can encourage a positive attitude towards school mathematics.

5. The Basic Characteristics of the Teaching/Learning Environment

Besides the use of suitable artifacts discussed above, the teaching/learning environment designed and implemented in the classroom, and present in all the studies we have conducted, is characterized by
• the application of a variety of complementary, integrated and interactive instructional techniques (involving children’s own written descriptions of the methods they use, whole-class discussion, and the drafting of a text by the whole class);

• an attempt to establish a new classroom culture also through new socio-mathematical norms, in the sense of Yackel and Cobb (1996).

As far as the first point is concerned, most of the lessons follow an instructional model consisting in the following sequence of classroom activities: a) a short introduction to the class as a whole; b) an individual written assignment where students explain the reasoning followed and strategy applied; c) a final whole-class discussion (comprising the clearer and more convincing explanations emerging from the whole-class discussion) aimed at socialization of the knowledge acquired. At the end of the teaching experiment the whole class contributes to producing a collective text. We consider that the interactivity of these instructional techniques is essential because of the opportunities to induce reflection as well as cognitive and metacognitive changes in students. This process may be very important for teachers also, since it enables them to recognize and analyze individual reasoning processes that are not always explicit (in the first phase, corresponding to the written report). In the second phase (corresponding to the collective discussion), comparing the different answers and strategies, children’s first attempts at generalizing, and further remarks made during the discussion, lead to collectively drawing up a collective text. This text, which completes the activity, is necessary to systematize the mathematical structures underlying the classroom activity; it is the phase of institutionalization of the mathematical concepts and processes shared by the whole class (Bonotto, 2005).

Concerning the second point, i.e. to establish a new classroom culture also through new socio-mathematical norms, the students are expected to approach an unfamiliar problem as a situation to be mathematized, not primarily to apply ready-made solution procedures, as in RME perspective. This does not mean that knowledge of solution procedures does not play a role, but the primary objective is to make sense of the problem. In practice, it will often be a matter of shuttling back and forth between the interpretation of the problem and a review of possible suitable procedures, models or results.

At the same time, the teacher is expected to encourage students to use their own methods, exploring their usefulness and soundness with regard to the problem. He/she is supposed to stimulate students to articulate and reflect on their personal beliefs, misconceptions and informal problem-solving strategies or models. By questioning assumptions and debating the relative merits of alternative strategies or models, the teacher and students negotiate and establish taken-as-shared meanings about the results that were more or less acceptable according to the situation (Bonotto, 2005). In other words, new norms about what counts as a good or acceptable response, or as a good or acceptable solution procedure were debated, in order to undermine some deeply rooted and counterproductive beliefs and attitudes such as a) mathematics problems have only one right answer, b) there is only one correct way to solve any mathematical problem (see Verschaffel et al., 1999).

According to the socio-constructivist perspective, these norms are not predetermined criteria introduced into the classroom from outside. Instead, the understandings are constructed and continually modified through the interaction between teacher and pupils, as well as by the artifact, whose introduction into the classroom setting brings from the outside world potential norms and ways of reflection that open lines of cultural conceptual development to the children. The development of mathematical reasoning and sense-making processes is seen as “inseparably interwoven with their participation in the interactive constitution of taken-as-shared (socio-)mathematical meanings and norms”, Yackel & Cobb (1996).

6. Some Results

The results of our teaching experiments show that, contrary to the practice of traditional word problem solving, children do not ignore the relevant and plausible aspects of reality, nor did they exclude real-world knowledge from their observation and reasoning. They confronted with this kind of activity also show flexibility in their reasoning processes by exploring, comparing, and selecting among different strategies (see e.g. Bonotto, 2003b and 2005).

These strategies are often sensitive to the context and number quantities involved, and closer to the procedures emerging from out-of-school mathematics practice; so mathematical reasoning
needed in extra-scholastic contexts, for example in work places, is favored (see e.g. Bonotto & Basso, 2001; Bonotto, 2003a and 2005). Finally also creativity and a problem critiquing process is favored; the children attempted to criticize and make suggestions or correct the problems created by their classmates or the results obtained (see e.g. Bonotto, 2006 and 2009; Bonotto & Baroni, 2008 and 2010).

Regarding the mathematical content we also laid the basis for overcoming some conceptual obstacles for example the misconception that multiplication always produces a larger result than the factors, an overgeneralization of a rule valid for integers. Several researchers have closely looked at the influence of the “multiplication makes bigger, division makes smaller” misconception in relation to the solution of word problems involving decimal numbers smaller than 1. When at the end of the experiment described in Bonotto (2005) it was asked during a whole-class discussion whether multiplication always produces results larger than the factors, the answer was, “It depends on numbers”, followed by the reason for that. This confirms the hypothesis that this kind of classroom activities can give support for accessing more formal mathematical knowledge and promote a process of “abstraction-as-construction”, in according to the “emergent modelling” perspective (Gravemeijer, 2007).

In another study (see Bonotto and Baroni, 2008) the artifact used is a story/cartoon strip in English and its protagonist is the class teacher. The only materials allowed were: a computer with Internet connection, cards with the exercise text and pencils. The personal computer was used as a special artefact because “The computer, as well as other more recent multimedia instruments, has a remarkable social and cultural impact and huge educational potential that perhaps has not yet been fully explored” (Bonotto, 2005).

One of the activity phases consisted in finding the cheapest travel mean for reaching Bratislava (by consulting websites suggested by the teacher). The activity only seems to be easy since Garda is very near to Verona airport as well as Venice airport and the three airports in Milan are easy to reach. This made it more difficult to calculate which route would be cheaper since the students had to calculate a series of possible combinations. We will provide an extract of a discussion, that followed the pairs work, as an example to show how these kind of open problems leave space for more authentic and realistic reflections compared to usual word problems.

Stefano: “The flight that leaves from Verona airport (return) costs a lot: € 395, 48. It costs 335,98 € from Milan Malpensa and so I will chose the second”.
Teacher: “do you all agree?”
Luca: “Actually, when I go away with my parents we always try not to leave from Milan because it is far away and if you go in a car the car parking costs a lot, otherwise you have to take a trains and then a bus”.
Federica: “Also, the flight from Milan is not direct and you need to change in Prague whilst Verona is a direct flight”.
Teacher: “Luca and Federica travel a lot with their parents and they have given us an excellent suggestion. If we decide to leave from Milan, we will also have to add the cost for reaching Malpensa. As Luca, who knows the Malpensa airport well, said you have to take a train from Peschiera to Milan and then a bus from the Train Station to the Airport. Do you still think it is the cheapest solution?”
Gaja: “I have just checked on the website that you told us about and the ticket from Peschiera del Garda to Milan costs € 7,60”.
Mirko: “It would actually be 2 x 7,60 € because you would also have to pay on the return journey... see the teacher is not going to stay in Bratislava because she needs to come back to school”.
Teacher: “Well done, I can see you are super travel experts! How much does the bus to the Malpensa airport cost?”
Mirko: “It costs € 8,00 return”
Teacher: “What would you suggest? Which is cheapest?”
Costanza: “Milan is the cheapest a 367,18 €... it is slightly cheaper than Verona that costs 395,48 €. Which do you want to book Miss?”
Teacher: “What do you suggest?”
Luca: “I have caught a plane from Malpensa a lot of times and I do not like it because it is far away and on the train it takes a long time to get there”.

Teacher: “You are right, even though the flight from Verona is slightly more expensive, I think it is better to leave from there because the airport is nearer and the flight is direct”.

After a few minutes, the students attempted to book a place on the plane online but one girl (the last one to connect to internet) raised her hand to point out a problem:

Veronica: “There is something the matter miss because my price is higher: it says 465, 98 €”

Teacher: “No, it is not a mistake! We are all trying to book a place on that flight. Does anyone understand what has happened?”

Mirko: “Perhaps there are no more places left!”

Teacher: “Actually, there are not all finished, just the seats that cost 395,48 €. What are you going to do?”

Veronica: “At this point it is cheaper for me to book the flight from Milan”.

Furthermore the use of the artifact brought about the evocation of situations that are in fact experienced, activating the ability both to pose and solve problems. For example, in a study in which the artifact used is a weekly TV guide (see Bonotto 2003b), one child tackled a spontaneous dilemma concerning a film whose review was next to the one assigned and whose viewing time partially overlapped. He posed a new type of question: “Once the first film is over, how much of the second film can I watch?”.

In another teaching experiment (see Bonotto and Baroni, 2008) the artifacts used were advertising leaflets for supermarkets, and the activities were carried out in pairs (chosen by the teacher so as to stimulate reciprocal support and assistance). The first part of the activity was exploration with the aim of investigating and interpreting the materials utilized. In this study we decided to propose a problem posing and problem critiquing activity. The idea that is the foundation of this activity is that there is a connection between problem solving and problem posing: it is impossible to solve any new problem without first having completely understood the exercise assigned by raising new problems during the solution phase, just as it is impossible to write the text of a problem without first having understood the mathematical area that is its foundation. The experience’s phases were:

1. the collective reading and analysis of the material: through collective discussion a meaning was given to the material and the numbers it contained;
2. each pair selected the problem’s data by extracting it from the material provided;
3. each pair created a problem containing the previously selected data: each problem had to contain a percentage calculation and two questions;
4. each pair had to resolve a problem written by another group;
5. through collective discussion, eventual errors or incongruences in the problems will be discussed.

We will show how the Problem posing activity and the subsequent Problem critiquing activity carried out together, in addition to having created interest and motivation, encouraged the children to create problematic situations that were both original and sometimes complex but nevertheless much more realistic in their content than word problems. This was all merit of contributions by the entire group who were called upon to write the final version of the texts together. For example, Sofia and Giorgio wrote:

“A mother sees that in a drawer there are several socks with holes and that all the pyjamas are too small. The following morning she looks at a calendar and realised that it is the start of the sales. She goes to a shopping centre and after looking long and hard she decides to buy three pairs of pyjamas at a full price of € 11,90 but with a 20% discount. How much will she spend on pyjamas? Subsequently she buys five pairs of socks that cost € 4,90 a pair and that have a 50% discount. How much will she spend on socks? If she has € 100,00 how much money will she have left?”

When the classmates were invited to give their contribution by adding, enhancing or changing the problem the following questions emerged:

“If the mother has run out of credit on her mobile phone and needs to call home urgently because she cannot remember her sons’ sizes, will she be able to recharge her phone? By how much?”

“If the supermarket’s car park costs € 1,50 per hour and the mother enters the supermarket at 10.30 and leaves at 12.00, how much will she spend? Will she be able to pay with the coins given to her as change from the supermarket or will she have to change banknotes?”
“When entering the shopping centre the mother sees that the following week all clothing will be discounted by 50%. How many pyjamas and socks can she buy?”

In another teaching experiment (see Bonotto and Baroni, 2010) the artifacts used were newspaper articles. In particular, certain scientific magazines suitable for children contain curious news items that often attract the children’s attention. The discussion that we will report took place in class when, after reading the text “A Flower Brunch” (Figure 1), the children were asked to work individually to find the data relative to the problem and compare their findings with those of the rest of the class. When the teacher asked them to report, one of the students triggered the following collective reflection.

Gabriele: “3 are the months necessary to put the flowers in the vases”.
Teacher: “So you entered 3 months in the data? What for?”
Massimo: “I don’t think that’s important, because there is no question that asks you the time”.
Mauro: “I don’t think so either. These aren’t problems, but newspapers, so it doesn’t work like in the problems in math books where all the numbers are necessary” […]

In the Italian curriculum and consequently in school textbooks, in consideration of the implicit didactic contract, the activities of reading and analyzing the problem data are seldom developed properly. The exercises present all the data necessary to solve the problem (unless the goal and consequently the chapter of the book is “Problems with unnecessary or insufficient data”). In this way the answer to the problem becomes in many cases nothing more than an exercise of the four operations and not a reflection on the data.

The following is an example that refers to the problem “Frio Cave” (Figure 2):

120,000 kg: 25 elephants = 4,800 kg

Mauro: “Teacher, we calculated that one elephant weighs 4,800 kg”.
Teacher: “Really?”
Fabiano: “Oh come on! That’s impossible! 4,800 kg isn’t nearly enough! It’s like saying 4.8 kg”.
Teacher: “Can someone explain that more clearly? How much is 4.8 kg?”
Raffaella: “It’s four kilos and eight hectograms”.
Sara: “Almost 5 kg”.
Teacher: “So?”
Mauro: “They’re right! It’s silly! It would be like saying an elephant weighs as much as my bookbag almost… Maybe I did the division wrong. I think I put a decimal point where I should have a comma for thousands”.
Teacher: “Good. Now we’re on the right track”.

As often happens, children forget the rule of multiplication with decimals and puts the decimal point in the wrong place.

The discussions are a clear example of how these artifacts and the news they contain are so close to the children’s life experience and that enables them to attract their attention and facilitate connection with their lives outside of school. It is this connection with reality that enables them to discern the students’ errors.

The results of our studies proved a fruitful one not only from the cognitive viewpoint but also from the metacognitive one. By presenting the students with activities that are meaningful because they involve the use of material familiar to them increased their motivation to learn even among the less able ones. A good example is the case of an immigrant child with learning difficulties related chiefly to linguistic problems. For her, as for many others, being confronted with a well-known everyday object with “few words and lots of numbers” acted as a stimulus. Indeed, it led her to say “It’s easier than the problems in the book because we already know how things work at a restaurant!” (Bonotto, 2006).

And in another teaching experiment a child said: “This is not a problem. Problems are full of words… and I can never do them because I do not understand very much. I can do these though because anyone can read prices on a flyer!” (Bonotto & Baroni, 2008).

This confirms that “Using a receipt, which is poor in words but rich in implicit meanings, overturns the usual buying and selling problem situation, which is often rich in words but poor in meaningful references” (Bonotto, 2005).
7. Conclusion

In this paper we have discussed the results of some teaching experiments based on the use of suitable cultural artifacts, interactive teaching methods, and the introduction of new socio-mathematical norms. An effort was made to create a substantially modified teaching/learning environment that focused on fostering a mindful approach towards realistic mathematical modeling and problem posing.

The results showed that, contrary to the practice of traditional word problem solving, children did not ignore the relevant, plausible, and familiar aspects of reality, nor did they exclude real-world knowledge from their observation and reasoning. Children exhibited flexibility in their reasoning processes by exploring different strategies, often sensitive to the context and quantities involved. Furthermore, by questioning assumptions and debating the relative merits of alternative strategies, the teacher and students negotiated and established taken-as-shared meanings about the results that were more or less acceptable according to the situation. In other words, new norms about what counts as a good or acceptable response were debated.

The positive results obtained in our teaching experiments can be attributed to a combination of closely linked factors: a) an extensive use of suitable artifacts that, with their incorporated mathematics, played a fundamental role in bringing students’ out-of-school meaningful reasoning and experiences into play, and allowed a good control of inferences and results; b) the application of a variety of complementary, integrated, and interactive instructional techniques; c) the introduction of particular socio-mathematical norms that played an important role in paying systematic attention to the nature of the problems and the classroom culture; d) an adequate balance between problem-posing and problem-solving activities, in order to promote also a mathematical modeling disposition.

“An innovative approach of problem solving, and the same is true for mathematical modeling depends ... on the courage to present complex situations and hard problems, some even unsolvable, to students, and, instead of asking for solutions, to listen to their proposals and to give space for their creativity to manifest” (D’Ambrosio, 2009).

We do not suggest that the activities of this project are a prototype for all classroom activities related to mathematics, although in agreement with other researchers we argue that the development of a certain kind of attitude towards mathematical problem-solving and problem-posing, and realistic mathematical modeling, should not emanate from a specific part of the curriculum but should permeate the entire curriculum.

But is there a reverse of the coin, if the word “reverse” can be used?

To implement this kind of classroom activities there needs to be a radical change on the part of teachers as well. They must try to i) modify their attitude to mathematics, which is influenced by the way they have learned it; ii) revise their beliefs about the role of everyday knowledge in mathematical problem solving; iii) see mathematics incorporated in the real world as a starting point for mathematical activities in the classroom, thus revising their current classroom practice; iv) know much more about the everyday of their students to offer them significant references to familiar and concrete situations belonging to the sociocultural environment of the pupils (Bonotto, 2009).

A learning environment like the one described here makes very high demands on the teacher, and therefore a revision and change in teacher training, both initially and through in-service programs, is necessary.
The largest bunch of flowers is composed of 100 kinds of different blooms. To realize it needed more than 3 months and its huge vase, hung at 7.5 meters from the ground, weighs more than 200 kg. It was made to inaugurate an Hotel in London. The inspiration is one of the seven world wonders... Babylonian gardens!

Figure 1: A flower Brunch

At Frio Cave more than 10 millions of bats take refuge everyday, so many to cover all the cave ceiling large as the Milan Cathedral. At the sunset, when they leave the cave, they need more than one hour to go out: They appear even in the USA aviation radar. Every night they eat about 120 Mg of insect, the weight of 25 elephants!

Figure 2: Frio Cave

References


Bonotto C., & Basso M. (2001). Is it possible to change the classroom activities in which we delegate the process of connecting mathematics with reality?. International Journal of Mathematics Education in Science and Technology, 32 (3), 385-399.


