Teachers’ conceptions of mathematical modelling at Swedish Upper Secondary school

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Abstract.
Researchers argue that teachers’ conceptions (beliefs) about mathematical modelling have an impact on the low integration of modelling activities into the mathematics classroom. This paper presents a case study of 18 teachers, investigating their conceptions about mathematical modelling and their experiences of working with modelling activities in Swedish upper secondary school. The results, based on an analysis with a grounded theory inspired coding strategy, indicate that the teachers’ conceptions of the notion mathematical modelling relates to designing a mathematical model based on a situation. It is also concluded that the teachers have minor experience of the notion of mathematical modelling in mathematics class, but in physics class modelling is used as a common activity. Overall the teachers in this study seem not to give priority to integrate mathematical modelling into their everyday mathematics teaching. One reason may be related to teachers’ conceptions about mathematics where more than every other teacher expressed that some of the modelling items discussed during the interviews were not considered to be mathematics.

Keywords: mathematical modelling, teachers’ conceptions, upper secondary school

1. Introduction

The research in mathematics education about mathematical modelling has gained interest during the last 50 years and has now reached maturation as a research discipline (Blum, Galbraith, & Niss, 2007). Arguments to include components of modelling into mathematics education were presented during late 60’s and modelling is now a part of many curricula around the world (ibid.). One of the countries with a curriculum emphasising the use of mathematical models and modelling is Sweden. The role of mathematical modelling in teaching and learning mathematics has been more explicitly described in the Swedish upper secondary school’s curriculum during the last 45 years (Ärlebäck, 2009a). In the present curriculum one can read that the use of mathematical models is one of four important aspects of mathematics, that should permeate all teaching (Skolverket, 2001). In addition, ”[t]he school in its teaching of mathematics should aim to ensure that pupils:… develop their ability to design, fine-tune and use mathematical models, as well as critically assess the conditions, opportunities and limitations of different models” (Skolverket, 2001, pp. 60-61). However, even if the notions of mathematical models and modelling are prescribed in the curriculum as central aspects of mathematics to be taught, there are no definitions of their meanings. This lack of definitions in the curriculum may open up for different interpretations about the notions among teachers, students, authors of national course tests, author of textbooks in mathematics and other persons related mathematics education.

There have been a few research studies in Sweden analysing students’, teachers’ and national course tests authors’ interpretations and/or experiences of mathematical models and modelling. For instance, an empirical investigation was performed by Frejd and Årlebäck (2010, 2011) with aim to explore how students describe the notions of mathematical models and modelling and how capable students are in solving modelling items. Some conclusions from the studies were that only 23% out of the 400 students from different parts of Sweden expressed that they had heard or used the notions in their education before and the students’ descriptions of the two notions were short in words (10 words in average). Possible reasons given by the authors were that the students had little or no experience of the notions, that the students did not have a clear view about them even if they have heard them or they had problems to describe and express their views in writing (Frejd & Årlebäck, 2010). Similar problems were identified in a qualitative study of two Swedish upper secondary teachers. The two teachers had difficulties during an interview to express their own conceptions about the notions of...
One activity familiar both to students and teachers is the national course tests in mathematics, which are assessing mathematical modelling as one competency out of six (problem solving, algorithm, concept, modelling, reasoning, and communicating). However, an analysis of test items by Frejd (2011) showed that aspects related to the intra-mathematical world primary were assessed, such as to calculate a result based on an already existing model. This may be a signal to the teachers, what aspects of the modelling process are important to teach, when considering Niss’ (1993) premise: “What is not assessed in education becomes invisible or unimportant” (p. 27).

One assumption described in research literature is that, teachers’ conceptions (beliefs) of mathematics and mathematics education is an essential factor for how they teach (Thompson, 1992). This is the case according to Kaiser (2006) who concluded that, teachers’ beliefs of modelling in mathematics education is an important reason for the low integration of modelling activities in mathematics class. The conclusion from Kaiser (2006) lead to the question, what conceptions do Swedish teachers’ have about the role of modelling in mathematics teaching. The aim of this paper is: to investigate teachers’ experiences of teaching mathematical models and modelling in upper secondary school and what conceptions they have about the notions of mathematical models and modelling.

The research questions posed to address the aim are:

1. What conceptions do teachers in upper secondary school express about the notion of mathematical modelling?
2. To what extent do they describe mathematical modelling activities as part of mathematics/mathematics education?”

The structure of this paper is first to clarify the research questions by discussing and defining the theoretical notions of mathematical modelling and conceptions. Then I will continue with explaining the methodology and finally the results will be provided and discussed.

2. Mathematical modelling and teacher conceptions

A comparative literature study by Frejd (2010) discusses three different perspectives on models and modelling in mathematics education with the aim “to be able to better analyze upper secondary school students’ descriptions of their own activities and thinking processes while engaging in some modelling activity and their views on the relevance and epistemological status of mathematical models in society” (p. 2). The paper by Frejd (2010) was mainly concerned with students’ in upper secondary school, but the discussions of methodological implications of adopting particular theoretical perspectives are in general terms and could also be relevant for analysing teachers’ descriptions of their activities and their conceptions about mathematical modelling. The three perspectives presented in Frejd (2010) were modelling as a cyclic process, modelling as a social process and a new perspective of the modelling process in terms of Anna Sfards’ theory of commognition. Modelling as a social process and modelling in terms of commognition stress aspects related to the teachers’ social situation. This study is an initial investigation and does not focus on the social aspects (i.e it takes teachers’ answers as direct indicators of their views). Therefore the chosen perspective for this study is modelling as a cyclic process. There are also other reasons for the choice, such as an interpretation of the Swedish curriculum by Palm et al. (2004) used as a base for the construction of national tests, which has a similar perspective on mathematical modelling as well as the test items in the “student questionnaire” used in this study (discussed in the method and methodology paragraph).

The perspective of modelling as a cyclic process (modelling cycle) is well known in mathematics education related to ICTMA\(^1\), often discussed in the literature (Blum & Niss, 1991; Blomhøj & Højgaard Jensen, 2003; Maaß, 2006; Borromeo Ferri, 2006; Kaiser & Sriraman, 2006; Blum & Leiß, 2007; Blum et al., 2007). There is not only one single description of the cyclic process but it may vary depending on the research aim (Borromeo Ferri, 2006). According to Kaiser et al.

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1. The International Community of Teachers of Mathematical Modelling and Applications (see web site at http://www.ictma.net/)
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(2006), the modelling process is described as a cyclic process with five up to seven sub processes and it involves two domains, one called reality and one called mathematics.

A short description of a cyclic process will be provided here with use of words from Blum and Leiß (2007). The starting point of investigation is a problem in the ‘real world’ called the real situation (the problem is often formulated in everyday knowledge). The modeller(s) or the problem solver(s) need to understand the task to make a mental representation of the situation (how the individuals are thinking about the problem situation), then continue to come up with a real model (external representations) by simplify/structure, filter and idealize the information. This real model is then moved from the ‘real world’ to ‘the mathematics world’ by mathematising these criteria and create a mathematical model. The final steps are to work mathematically in the “mathematical world” to produce answers, mathematical results, to interpret the mathematical results into real results by moving back to the ‘real world’ and to validate the real results. If the validation of the real results seems to be incorrect and other aspects need to be included, then the modeller has to do the cyclic process over again.

The use of modelling competency adapted in this paper relates to the entire process of modelling or as Blomhøj and Højgaard Jensen (2003) define it “[b]y mathematical modelling competence we mean being able to autonomously and insightfully carry through all aspects of a mathematical modelling process in a certain context” (p. 126).

Using the modelling cycle in investigations of teachers’ thoughts about mathematical modelling it requires a methodological discussion about thinking processes (Frejd, 2010). In this study the notion of conceptions and beliefs need to be discussed.

The word beliefs in educational research in mathematics is widely used. There are books, chapters and articles discussing this notion, such as Thompson (1992), Leder, Pehkonen and Törner, (2002) and Philipp (2007). However, the definition of beliefs is not explicitly described in many studies and the reader is expected to know what it means (Thompson, 1992). There does not exist a general agreement on a single definition in the literature (McLeod & McLeod, 2002) and according to Törner (2002) “[i]t is clear that only in rare cases can a final precise definition of all components of a belief definition be achieved in a specific context” (p. 91).

The word conceptions has also been used with different meanings in mathematics education (Furinghetti & Pehkonen, 2002). There are researchers who argue that conception is a broader notion and see beliefs as a part of conception (Thompson, 1992; Philipp, 2007). Others claim a clear distinction between conception and beliefs, see for instance Furinghetti and Pehkonen (2002) discussing the issue.

To illustrate the difference between beliefs and conception I will use the definitions from Philipp (2007, p. 259).

**Beliefs-** Psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt less intensely, and are harder to change than attitudes. Beliefs might be thought of as lenses that affect one’s view of some aspect of the world or as dispositions toward action. Beliefs unlike knowledge, may be held with varying degrees of conviction and are not consensual. Beliefs are more cognitive than emotions and attitudes. (I do not intend this definition under affect because, although beliefs are considered a component of affect by those studying affect, they are not seen in this way by most who study teachers’ beliefs.)

**Conception-** a general notion or mental structure encompassing, beliefs, meanings, concepts, propositions, rules, mental images and preferences.

Looking at Philipp’s definition of beliefs above, one can read that beliefs can be held with varying degree of uncertainty. However, if beliefs are held with certainty then they are called knowledge (Philipp, 2007). This implies that the researcher should consider making a distinction between knowledge and beliefs, at least if the research aims to investigate possible changes in teacher behaviour. Beliefs are possible to change, but knowledge is not (Furinghetti & Pehkonen, 2002). The debate about the distinction between beliefs and knowledge as well as if it useful will continue among researchers (Philipp, 2007). The definition of conception above involves beliefs, but also other notions like meanings, rules and preferences. Conception is therefore broader in the sense that the researcher does not distinguish between beliefs and preferences or beliefs and mental images etc.
However the notion of knowledge in the definition above might only be seen implicit. The debate about the relations between conception and knowledge could be similar to the debate about the relations between beliefs and knowledge.

There is not much research done about teachers’ conceptions (beliefs) of mathematical modelling in Sweden. The present study is an exploratory investigation with an aim to find some first indication of teachers’ conceptions in a broad sense about mathematical modelling. Therefore the distinction between knowledge and conception/beliefs is not necessary and I have adopted the definition of conception by Lloyd and Wilson (1998) “[w]e use the word conceptions to refer to a person’s general mental structures that encompass knowledge, beliefs, understandings, preferences, and views” (p.249).

There have been a few studies related to teachers’ beliefs about mathematical modelling in mathematics education. Kaiser (2006) did an empirical investigation and “it became clear that although teachers were convinced to considering applications and modelling for daily school practice they still argued for mathematics and mathematics teaching in which application and modelling only played a minor role” (p. 393). In addition, Kaiser and Maass (2007) also stressed that mathematical modelling only played a minor role in teachers’ beliefs about mathematics and mathematics education. The case study by Ärlebäck (2010), discussed in the introduction, is so far the only one in Sweden addressing the issue. However, the present study, which will be explained in more detail in the next section, will provide more Swedish results on teachers’ conceptions.

3. Method and methodology

Frejd and Ärlebäck (2010), as mentioned in the introduction, used a ‘student questionnaire’. The ‘student questionnaire’, which is used also in this study, included an open question, seven test items and some follow up questions. The open question was used to find students’ descriptions of the notions of mathematical models and modelling and the follow up questions concerning attitudes relating to mathematical modelling, previous experiences of mathematical modelling, gender, grade etc. The seven test items originated from Haines et al. (2000) and are multiple choice questions, which have been used in different research studies (Haines and Crouch, 2001; Izard et al., 2003; Ikeda, Stephens, & Matsuzaki, 2007; Lingefjärd & Holmquist, 2005; Kaiser, 2007). According to Haines et al. (2000), “it is possible to obtain a snapshot of students’ [modelling] skills at key developmental stages without the student carrying out a complete modelling exercise” (p.10) by the use of these questions. A methodological concern in Frejd (2010) using the modelling cycle is the distinction between the two domains ‘reality’ and ‘mathematics’. However, the test items acknowledge a more or less clear distinction between the two domains, due to assessing different stages in the modelling cycle. An example of a test item is provided below.

**Figure 1.** Item nr 1 from Haines at al. (2000)
A translated version of the test item in Figure 1. was used in the student questionnaire and is aiming to assess the competence level of a transition from the real situation to the real model (formulating a model). The student questionnaires were delivered by research students (FontD) during the spring of 2009 to 41 classes across Sweden. In addition to the student questionnaires a teacher questionnaire was attached to all mathematic teachers that superintended the survey, with questions about gender, years of working experience as teachers, the name of the school, if they had heard the notion of mathematical modelling before, if they could describe the meaning of the notion as well as if they were working with modelling activities and how. The purpose of these teacher questionnaires was to provide some initial information about teachers’ views of mathematical modelling and to identify the schools that were participating in the study to investigate differences between how modelling activities are used in different geographical regions.

There were 18 teachers (three females and 15 males) from 12 different schools across Sweden who responded to the questionnaire. The teachers had a teaching experience between 2 to 30 years and three of the teachers had also some research experience from mathematics or mathematics education. However, all teachers did not answer all questions and the received answers were often expressed in short sentences with only little facts. The teachers had only described only a few examples of modelling activities. There might be several reasons for the lack of answers. It may be due to the design of the questionnaire with only one page, the teachers did not have the time to fill in the form, the notion of mathematical modelling is not familiar to the teachers. However, in order to find possible answers to my research questions as well as to find similarities/differences between the teachers’ descriptions and the students’ descriptions about the notion of mathematical modelling in Frejd and Årlebäck (2010), an interview investigation was planned with the teachers who responded to the questionnaires. This approach is in line with Robson (2002), who argues that one appropriate circumstance for using an interview in qualitative research is to investigate about meanings of a certain phenomena. The decision of an interview study raised several methodological questions, such as:

i) What ethical considerations must be taken in order to get in contact with the teachers when they have answered a questionnaire anonymously?

ii) What options are there to do interviews when the participants are spread around Sweden?

iii) How should the interview guide be designed?

The first question (i) may be seen as an ethical dilemma since on the one hand the teachers have answered the questionnaire anonymously, but on the other hand I as a researcher have access to the name of their school and their working experience, which means that I might have the possibility to identify them. To overcome the dilemma I asked all research students who delivered the questionnaires to ask the participating teachers if I was allowed to contact them. All the 18 teachers agreed to be contacted and their names were given to me by the research students. To the second question (ii) both Robson (2002) and Bryman (2004) suggest telephone interviews where money and time is a restriction for face-to-face interview. In addition Bryman (2004) suggests online interview by e-mail. Based on the experience from the initial teacher questionnaire where teachers had to formulate themselves in written text and on Bryman’s (2004) concerns that respondents more likely drop out during the exchange, I chose the telephone interview.

The final question (iii) was answered by using suggestions by Kvale (1997) in his third stage Interviewing in the seven stages of an interview investigation (Thematizing, Designing, Interviewing, Transcribing, Analyzing, Verifying, and Reporting). In the Interviewing stage it is suitable to develop two interview guides one with the research questions and one with the interview questions. The interview questions should pay attention to both the aim and research questions as well as the dynamic flow of the interview (Kvale, 1997). The interview guide in this study, which can be found in the appendix, was developed based on these conditions together with the idea to visualize how the research questions and the interview questions are related. To develop the interview guide, the two research questions have been divided into seven auxiliary questions (A-G, see appendix) to make the transition from research question to interview question more explicit. The first research question ‘What conceptions do teachers in upper secondary school express about the notion of mathematical modelling?’ is equal to the first auxiliary question A focusing on the notion, the second research

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2 The Swedish National Graduate School in Science, Technology and Mathematics Education Research. (see web site at http://www.isv.liu.se/fontd/nationella_fontd/start/1.180370/FolderAllmn_web.pdf)
question “To what extent do they describe mathematical modelling activities as part of mathematics/ mathematics education?” is depending on the outcome of A and covers a more wide range of possible approaches that could be taken in considerations, here illustrated as auxiliary questions B-F. The auxiliary question B refers to teachers’ conceptions of mathematics in general, C refers to teachers’ conceptions about teaching mathematical modelling, D refers to teachers’ conceptions about different parts in the modelling process, E refers to teachers conceptions of modelling in relation to tests, course literatures and other activities in the class room, and F refers to teachers’ conceptions of the test items used in Frejd and Årlebäck (2011).

The first interview question, What courses and programs do you work with?, together with the sub question, for how long have you been working as a teacher?, was not designed to be related to any of the auxiliary questions. Instead it was used as a warm up question as a non-threatening initial question (Robson, 2002) to establish background information of the participating teachers. The second interview question, Why did you become a teacher in mathematics? together with the sub questions, what is your main goal with teaching? and what use of mathematics do you see as important for students to learn?, I aimed to find some expressed conceptions from the teachers of mathematics in general. There were a few citations from the official curriculum guidelines about mathematical modelling in the student questionnaires in Frejd and Årlebäck (2011) and the same citations were found in the teacher questionnaire. These citations may cause an impact on the third Interview question, what is your interpretation of the notion of mathematical modelling? This interview question is related to the auxiliary questions A (Their answer may indicate some conceptions of the notion), D (Their answer may indicate conceptions of different parts of the modelling process) and E (Their answer may indicate if their views refer to official/local curriculum document). The following four interview questions (4-7) and corresponding sub-questions were developed by using the same strategy to find relations and connections to the auxiliary questions (see Appendix), in order to receive possible answers to the research questions.

A pilot interview took place in spring 2010 with one of the 18 teachers who is situated close to the researcher. He was given the seven interview questions in advance along with a student questionnaire and the instructions that the test items in the student questionnaire and the interview questions were going to be discussed during the interview. Besides to test the interview questions, another condition to be tested during the pilot was the time of the interview. The teachers are busy in the end of the semester due to for example national course test, grading, evaluations. To convince all the teachers to participate in the study the interview had to be short in time. The pilot interview took about 17 minutes and after the interview the teacher was asked to critically comment the questions. No negative comments or attitudes were expressed by the teacher and the interview questions seemed to be operational. All the other 17 teachers agreed to be interviewed after having been contacted by telephone, where I described the study and the time length to be about 20 minutes. An e-mail was sent after the phone call with the interview questions, a student questionnaire and some general information about the study and an opportunity to choose a time for the interview. By sending the interview questions as well as the questionnaire in advance may have both positive and negative consequences. It is positive that the teachers have access to the information about aim and interview questions, which is important for their consent. The teachers also have more time to think about the answers and the interviewer may have less influence on the interview, which may give a more coherent picture. Negative consequences are that the teachers may prepare themselves differently, as some may search on the web, discuss with colleagues, get influenced by the test items, etc, which could have an impact on their conceptions on mathematical modelling. However, even if they have not prepare themselves they will get some time during the interview and for those who have done a deeper preparation it may show their interest in discussing the interview questions. The positive effect of sending the interview questions as well as the questionnaire in advance seemed to be in prior. Also the fact that the teachers had probably seen the test items before in the study by Frejd and Årlebäck (2011) made the decision easier.

Before each interview the teacher was asked for permission to record the interview, which all teachers accepted. The interview process was then carried out by the interview questions, sub questions and other follow up questions in a flexible manner, i.e. semi structured interview (Robson, 2002; Bryman, 2004). The semi structured interviews were transcribed and coded with a coding strategy inspired from grounded theory, which will be discussed in the next section.
4. Analysis

Altogether there were 189 transcribed pages (12p, 1.5 line spacing) which were analysed. The choice of scrutinizing the data into different codes by using a grounded theory inspired approach of the coding procedure is twofold: 1. the study is an exploratory study, which means to build a proposal about teachers’ views rather than test an already existing one and to handle much data in a systematic creative way to be able to identify, relate and develop alternative meanings of the phenomenon. This is in line with Strauss and Corbin (1998) coding procedure; 2. a similar procedure was operational in the (related) research study by Frejd and Årlebäck (2010) in order to analyse students written expressions about the notion of mathematical modelling and thus it might be useful for this study as well. The inspired approach for developing coding categories originates from the process of open coding and axial coding (Strauss & Corbin, 1998). The open coding is the process where the data have been split into discrete parts in order to develop initial categories by examining and comparing the discrete parts. The process of identifying relations, conditions and interactions between the open categories and linking the categories together is called axial coding. To illustrate this procedure, I will in the following two sections give examples of how the coding has been done.

4.1. Open coding

Each answer (coding unit) to every interview question has been examined closely using a “line-by-line analysis” (Strauss & Corbin, 1998, p. 57), by asking to the data what is essential in this answer, what it means. Nine examples (three examples of three questions) are given below (my translations) to describe the open coding procedure:

Example 1 [interview question nr 2]

Interviewer: Why did you become a teacher?
Teacher K: Well I do not have a really good answer, but my mother was a teacher and I had planned to be an engineer, but it felt like the labour market at that time was concentrated on construction and military. I have always been interested in mathematics and physics and people have told me that I was good at explaining before I became a teacher and I also have friends who became teachers, so there where different reasons.

This coding unit above (example 1) has been categorized into the following (why did you become a teacher) categories: (nr 13) I do not have good answer/difficult question; (nr 14) I have relatives and/or friends whom are teachers; (nr 15) did not want to become something else, like a military; (nr 5) mathematics is fun or interesting; and (nr 16) I had got information to be a good teacher.

Example 2 [interview question nr 2]

Interviewer: Why did you become a teacher?
Teacher D: There are two reasons; first I have had the favour of having a gift for mathematics, chemistry and physics. Not biology, there you had to learn too much by heart and even today I do not recognize that as knowledge. To know something is one thing, but to understand something is knowledge.

Interviewer: Yes
Teacher D: That was one thing that I found mathematics to be easy and the other which might be even more important is that I had a fantastic, fantastic mathematics teacher in junior secondary school who could explain. And then I got a senior lecturer at Chalmers University [name], and it did not get worse.

Interviewer: Right.
Teacher D: Then I got a push forward and I got ideas on how to teach, which I have used.
The following open coding categories have been used in process in order to categorize example 2: (nr 3) I have good knowledge in mathematics or I have a gift for mathematics; (nr 8) I’m inspired by a teacher; (nr 9) I have my own ideas on how to teach; and (nr 24) Mathematics is a special kind of knowledge.

Example 3 [interview question nr 2]

Interviewer: Why did you become a teacher?
Teacher P: Yes, well, say that, well, I think I wanted to find a good, free and varied job, where you meet people especially youths.

Interviewer: Yes, right.
Teacher P: And then, I have been interested in mathematics and Physics and felt it would be fun to work with youths.

The coding unit above (Example 3) is categorized as: (nr 13) Am a bit unsure; (nr 22) free and varied job; (nr 20) Want to work with (young) people; and (nr 5) mathematics is fun or interesting.

Example 4 [interview question nr 3]

Interviewer: What meaning do you give to the notion of mathematical modelling?
Teacher M: Yes, it is to describe something with a mathematical text, with mathematical symbols, an actual course maybe like in physics like I do. To describe a phenomena, to design a model of something visible.

Interviewer: Yes
Teacher M: To describe it with figures and formulas, I think it is more in physics where you encounter this as a teacher.

The open coding process for the coding unit above have turned into the following (meaning of modelling) categories: (nr 2) to describe ‘something’ with mathematics; (nr 3) a connection to working situations in natural science; (nr 8) to design a model with mathematics; and (nr 12) related to formula, variable etc.

Example 5 [interview question nr 3]

Interviewer: What meaning do you give to the notion of mathematical modelling?
Teacher F: What I think about when doing mathematics, is not just to calculate. To calculate is an ability which to a large extent is possible to automatize and does not really demand much intelligence. However, the big challenge is often to translate everyday-problems or any problems to a symbolic language. That is what I think is to show mathematical intelligence. Einstein writes in his equations that he doesn’t know how to solve them, but he is excellent to express in symbolic language. That is what I interpreted as mathematical modelling. When you have written them down, it often easy to calculate with them, but it demands a whole lot of creativity to write them down.

Example 5 has been categorized as (nr 5) focus on the transfer between different discourses; and (nr 7) relate to problem solving activities.

Example 6 [interview question nr 3]

Interviewer: So, then you had not heard the notion before?
Teacher R: Not explicitly the expression mathematical modelling.

Interviewer: precisely.
Teacher R: I was sitting here wondering what it means, maybe one use to bring this up in contexts where a model is found for something. One has
created/designed a function or connection and it is not valid outside. It gets crazy, this model only works in a small interval.

Interviewer: Yes.
Teacher R: One brings up some of these things when one looks at task in the textbooks. Tasks in the textbook that include designing a formula, build up a function or a function to look at.

The coding unit above (Example 6) has identified to include the following open coding categories: (nr 8) to design (a function as) a model for something; (nr 11) connection to functions; (nr 12) connection to formulas; and (nr 13) validate a function.

Example 7 [related to interview question nr 6. Analyzing reasons of Why do you use modelling activities?]

Teacher B: It is hard to work without the use of mathematical modelling in mathematics, I think. Then it is another thing if we call it the right name.

The question, why do you use modelling activities?, was not explicitly asked to teacher B. However teacher B’s answer, the coding unit above (Example 7), was used in the open coding procedure to categorize reasons related to the sub-question. The category used was (nr 16), it is difficult to do mathematics without mathematical modelling.

Example 8 [related to interview question nr 6. Analyzing reasons of Why do you use modelling activities?]

Teacher F: By the interpretation I gave, so is it a way of working, which is quite nice. I try to find problems with that character to make students to think. I often do a giant problem.

Reasons for doing modelling activities in the coding unit (example 8) were characterised in the following categories: (nr 3) a way of working; and (nr 5) make students think.

Example 9 [related to interview question nr 6. Analyzing reasons of Why do you use modelling activities?]

Teacher O: No, as soon there is a moment in the curriculum where you can apply mathematical models to reality, then I’m quite clear that the students shall translate real course of event to mathematical models. They have to do regular things too, they have to do equation solving and percent solving and cannot work with models all the time. Even if there is percent, so is it possible to work with exponential functions and so.

This coding unit (Example 9) is used to categorize “Why do you use modeling activities” and the following open coding categories have been used: (nr 11) translations between different discourses (course of event to mathematical models); (nr 12) to practice on mathematical modelling (according to teacher O is modelling to go between different discourses); and (nr 4) to show the connection to reality.

4.2. Axial coding

To link the categories together required considerations concerning on how to identify the diversity of conditions associated with each open coding category and how to relate and reassemble
the categories into new broader categories to develop axial coding categories. The example related to the interview question nr 7 will be used to explain the axial coding process. I identified 31 open categories related to the issue, why would you not use all the test items in mathematics class (see Appendix for all the 31 categories). Questions were asked on ‘how’, but also ‘why’ the open coding categories were related. And the final axial coding categories are displayed in the table below.

### Table 1. The final axial coding categories related to interview question nr 7.

<table>
<thead>
<tr>
<th>Axial coding categories</th>
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<tr>
<td>A. Problems in change of teaching</td>
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<tr>
<td>B. No mathematics</td>
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<tr>
<td>C. Not a demand</td>
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<tr>
<td>D. General reasoning</td>
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<tr>
<td>E. No experience or don’t know</td>
</tr>
<tr>
<td>F. Problems related the test items</td>
</tr>
<tr>
<td>G. Other subjects</td>
</tr>
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The reasons for the construction of the final axial coding categories as viewed in Table 1 are:

**Category A:** Some of the open coding categories seemed to be related to problems in changing current teaching and these arguments have turned into the axial coding category A. The following open coding categories are Category A:

(nr 8) No time;
(nr 9) Not for fun [“...they need to have some mathematical tool as a part of the ongoing course else there is no time, we cannot use them as pure entertainment”, teacher C];
(nr 1) Course literature, (nr 2) students lose pace [ “... I have noticed in A and B course in upper secondary school a resistance to leave the books ... they feel that they lose the pace” teacher A];
(nr 26) Pre-knowledge [“I do not think it is easy to start on this level without practising on a more basic level and then increased the difficulties” teacher M];
(nr 28) Extra [“everything extra, will not be done”, teacher N];
(nr 31) Not interesting for the students [“..., but the question is if the students would find the items interesting”, teacher R].

**Category B:** Many of the teachers explicitly mentioned the lack of mathematics in relation to some of the test items, and the following open coding categories are included in category B:

(nr 4) No mathematics [“Those in the beginning,..., do not have so much to do with mathematics” teacher E],
(nr 16) Discussed outside mathematics [“this is an item that you may discuss in other situations than in mathematics” teacher F],
(nr 17) No mathematical thinking [ “it is easier to get the student to think in mathematical terms in the last three items” teacher F ]

**Category C:** This coding category focuses on arguments related to curriculum (course) documents and demands from universities and the open coding categories included are:

(nr 7) No connection to the course [“...they need to have some mathematical tool as a part of the ongoing course else there is no time”, teacher C].
(nr 23) No demands from university [“And if one thinks of the demands from higher education (university) I think we cover them with traditional teaching”, teacher K],
(nr 27) No goal [ “not if you will fulfil the program goals...”, teacher N].

**Category D** captures the aspect of general reasoning expressed by several teachers. The following open coding categories have been included in Category D:

(nr 5) Common sense [Item 1 “would not be seen as mathematics, maybe it would be related to common sense”, teacher M],
(nr 13) Time killer [“What shall I say, some of the items have a character of time killers and common sense.”, teacher E].
(nr 14) Logic ["Item one for instance where the mathematics not obvious it is more logical thinking maybe", teacher K],
(nr 29) Multiple choice test for higher education ["...it has characters of the test for higher education..., which includes logic", teacher Q].
Category E includes teachers expressions about lack of experience of the test items before, which may be one reason for not knowing if it is useful or not. The following categories have identified to be part of Category E:
(nr 3) Do not know ["It is hard to say [if one should use the item in mathematics class] but it is never wrong to discuss different alternatives, aa, but I do not really know", teacher J],
(nr 12) New type of problem ["The issue in the items is quite new", teacher E], (nr 18) Have not thought about it ["...I can understand way, but I have not thought about it", teacher G],
(nr 20) Lack of experience ["items 1 and 5 are also interesting, but it is nothing I have done before", teacher I]

Category F relates to problems the teachers expressed concerning the test items. Categories are:
(nr 6) More conditions needed ["...they (items 1 and 2) can gain more power if you put in more conditions like you did in some other item", teacher B],
(nr 10) Difficult to understand ["When I think on how I put items together to students, then I have a feeling that one has difficulties to understand these items", teacher C],
(nr 11) No probability, (nr 32) Nothing to measure ["...we cannot use them as pure entertainment, they need to include something you can measure or some probability", teacher C],
(nr 19) Items are difficult to discuss ["Item 5... is not so easy to discuss" "teacher G],
(nr 22) Open problems are difficult ["...I don't know I think it is hard with theses open problems", teacher J],
(nr 24) I do not understand ["I do not understand them really", teacher K],
(nr 25) Text ["I think that many of our students would have problems to grip the content, it is a lot of text", teacher M]

Category G: Relation to test items in other subjects are identified by the following open coding categories:
(nr 21) Philosophy ["I think they (the items) feel more philosophical", teacher J], (nr 30) It is discussed in physics ["You do this in physics when you have a condition and the students are suppose to find a connection without knowing the factors, quite similar to test item 2", teacher Q].

4.3. Reliability

The inter-observer reliability is agreements obtain between two or more independent coders which have coded the same material (Robson, 2002). To evaluate a coding procedure when there is a 'single-observer', Robson (2002) suggests “to enlist the help of a colleague who will devote the time necessary to learn to use the schedule and observe a proportion of the sessions to be coded” (p. 340). To analyse the inter-reliability of the coding procedure in this study a second independent researcher was asked to code two questions. The second researcher was given the instructions to code the data with the use of the open coding categories to interview question 2 (Why did you become a teacher in mathematics?) and the axial coding categories to interview question 3 (What is your interpretation of the notion of mathematical modelling?). These two questions were chosen to represent a general picture of the coding procedure. The interview question 2 consisted of 23 different open coding codes. This is more than the average number of open coding categories in this study, which is 17 open coding categories per question. However, in this data set the 23 different codes have an ordinary distribution of open coding codes and may be a good base for an external coding evaluation. The meaning of ordinary distribution in this paper is the frequency of codes per category and this frequency should mainly include one to five codes per category except for a few categories with more codes. The axial
coding categories for question 3 are representative in the sense that it contained of six different coding categories, which is the most frequent number of axial coding categories in this study. The question 3 is also important in this paper, because it is explicitly connected to research question one (What conceptions do teachers in upper secondary school express about the notion of mathematical modelling?) and therefore a good choice for an external axial coding evaluation.

For discussing reliability Holsti’s method (Holsti, 1969) was used. (i.e. a formula for computing reliability (R)). Holsti’s method is displayed below:

<table>
<thead>
<tr>
<th>Holsti’s method for reliability (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ R = \frac{2C_{12}}{C_1 + C_2} ]</td>
</tr>
</tbody>
</table>

\( C_{12} \) is the number of codes assigned and agreed upon by both coders. \( C_1 + C_2 \) is the total number of codes assigned by both coders.

According to Kaid and Wadsworth (1989) an \( R \)-value above 0.85 is satisfactory. If the \( R \)-value value is below 0.80, the researcher should react and make suitable changes (Kaid & Wadsworth, 1989).

The calculated \( R \)-values in this paper for the open coding process was 0.83 (\( C_{12} = 43, C_1 + C_2 = 104 \)) and for the axial coding process was 0.83 (\( C_{12} = 19, C_1 + C_2 = 46 \)). As seen above both \( R \)-values are neither above 0.85 nor below 0.80. One issue that came up during the external coding was that some of the categories could have been more clearly explained. The instructions only included the categories and no explicit example with excerpts were provided. A better instruction might have provided a better result. However, no further actions were taken because the \( R \)-values were close to 0.85 and not below 0.80.

5. Results

The result section will first present some background information about the participating teachers and then present the result from the 7 interview questions. All the 18 upper secondary teachers from different parts of Sweden had a teaching experience with most of the mathematics courses in upper secondary school. A majority of the teachers (16 out of 18) were working with the natural science program in 2009 or 2010. The number of working years as a teacher is displayed in the frequency diagram in Figure 1.
for students to learn” (16 open to 8 axial) and “what is your main goal with teaching”(11 open to 7 axial).

### Table 2. Results related to interview question nr 2.

<table>
<thead>
<tr>
<th>Why did you become a mathematics teacher?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Positive comments about mathematics</td>
</tr>
<tr>
<td>2. Positive towards teaching</td>
</tr>
<tr>
<td>3. Change in current situation</td>
</tr>
<tr>
<td>4. Were inspired of someone</td>
</tr>
<tr>
<td>5. No active choice</td>
</tr>
<tr>
<td>6. Good content knowledge</td>
</tr>
<tr>
<td>7. Positive towards working with people</td>
</tr>
<tr>
<td>8. Found beneficial parts of teaching</td>
</tr>
<tr>
<td>9. Difficult question</td>
</tr>
<tr>
<td>10. Always have known</td>
</tr>
<tr>
<td>11. To increase students’ understanding</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>What use of mathematics do you see as important for students to learn?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. “Everyday mathematics”</td>
</tr>
<tr>
<td>2. In education and science</td>
</tr>
<tr>
<td>3. Cognitive aspects</td>
</tr>
<tr>
<td>4. All mathematics is not useful</td>
</tr>
<tr>
<td>5. For future work and higher salary</td>
</tr>
<tr>
<td>6. As a tool in general</td>
</tr>
<tr>
<td>7. To picture mathematics as comprehensive</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>What is your main goal with teaching?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. To teach mathematics</td>
</tr>
<tr>
<td>2. To get students interested in mathematics</td>
</tr>
<tr>
<td>3. Difficult question</td>
</tr>
<tr>
<td>4. To get students to understand mathematics</td>
</tr>
<tr>
<td>5. To see the use of using mathematics</td>
</tr>
<tr>
<td>6. Cognitive and self confidence aspects</td>
</tr>
<tr>
<td>7. Economy</td>
</tr>
</tbody>
</table>

The main expressed argument for becoming a teacher (see Table 2) is positive comments about mathematics which includes statements from the teachers like “I enjoy mathematics”, ’mathematics is fun or interesting’. In addition, positive towards teaching (stimulating to teach, teaching experience, own ideas on how to teach), change in current situation (a change of working career, a change in education) and inspired of someone (inspired by another person) are also frequently mentioned. The most important factors for learning mathematics according to the teachers (see Table 2) are everyday mathematics or mathematics outside school such as in life, in society as general knowledge and in education and science (for further studies, in other subjects, scientific language). Cognitive aspects (practice the brain, logic reasoning) are expressed by seven out of 18 teachers to be important factors for the students to learn mathematics. The main goals with teaching are expressed in terms of teach mathematics (to teach mathematical content), get the students interested in mathematics and get students to understand mathematics (to facilitate for learning and understanding of mathematics). However, this question seemed to be a difficult question, seven out of 18 teachers explicitly answered that it was a difficult question or they did not know.

The third interview question investigates teachers’ responses of how they describe the notion of mathematical modelling. Only 50% of the teachers expressed that they had heard the notion before participating in the study of Frejd and Årlebäck (2010). The six axial coding categories as displayed in Table 3 below were developed from 13 open coding categories.

### Table 3. Results related to interview question nr 3.

<table>
<thead>
<tr>
<th>Descriptions of the notion of mathematical modelling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. To describe or simplify “something”</td>
</tr>
<tr>
<td>2. To go between different discourses</td>
</tr>
<tr>
<td>3. Problem solving</td>
</tr>
<tr>
<td>4. Relation to natural science</td>
</tr>
<tr>
<td>5. Validation of a model</td>
</tr>
<tr>
<td>6. Related to the quotations in the curriculum</td>
</tr>
</tbody>
</table>

A large proportion of the teachers (see Table 3) associated mathematical modelling to describe or simplify “something. Where the word something is described in terms like course of event, reality,
event, connection, something visible and things. Three examples of teachers’ descriptions are provided in the analysis section “open coding” and below are more examples from the teachers:

Teacher C: Well, you should find a, what shall I say, a mathematical function describing a course of events.

Teacher G: My interpretation is, well, you have reality, then you have a, what shall I say, you find a connection for it. It may happen if I get more time to think, I might do another interpretation.

Teacher I: I have never thought of what the exact meaning is, so my interpretation might be blurry, one may think that it is simply to describe some event with mathematics. It is described in a broad sense, it could mean anything.

Interviewer: Yes, what do you mean with an event?
Teacher I: An event can be, physical, stock market, you mean anything, like the items like that you have, supermarket checkout lines and such like.

Teacher P: It is, maybe, in mathematical terms describe a course of event.
Interviewer: Yes, right
Teacher P: Something you do not have a given formula for, instead to find something that can be described, described in mathematical terms that you want to describe.

Teacher Q: I’m not really sure.
Interviewer: No
Teacher Q: but
Interviewer: If you would try to do an interpretation.
Teacher Q: Because, I also have physics and all theories in physics are mathematical models of reality, well I have thought of something similar.

In the fourth interview question 10 teachers agreed that they had a local curriculum at their school and 6 out of them expressed that the local curriculum document were either the same as the official curriculum or a formal document, which is not used in practice. None of the local curricula had the notion of mathematical modelling described or mentioned.

Do you use modelling in your teaching? was the fifth interview question. There were eight teachers who expressed that they were not working with mathematical modelling in upper secondary school. Consequently, ten teachers expressed that they work with mathematical modelling, five out of those ten described that they were working with modelling intentionally, two out of those ten stated it was not intentionally and the rest (three persons) hesitated on the question. Questions were also asked on why or why not they use modelling in their teaching. Table 4 next page shows the final axial coding categories (why 10 open to 6 axial and why not 6 open to 4 axial) related to these questions.

<table>
<thead>
<tr>
<th>Why do you use modelling in your teaching?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. To show how mathematics is used outside school</td>
</tr>
<tr>
<td>2. A way of working</td>
</tr>
<tr>
<td>3. Make students think</td>
</tr>
<tr>
<td>4. To practise modelling (models)</td>
</tr>
<tr>
<td>5. I do not know</td>
</tr>
<tr>
<td>6. To practice on the transfer between different discourses</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Why do you not use modelling in your teaching?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Have not reflected on why not</td>
</tr>
<tr>
<td>2. Belongs to physics</td>
</tr>
<tr>
<td>3. Takes too much time</td>
</tr>
<tr>
<td>4. Students need to work on basic skills</td>
</tr>
</tbody>
</table>
As already mentioned, there were five teachers who argued that they used modelling intentionally in their teaching. The two main arguments the five teachers used (axial coding categories) are to show how mathematics is used outside school and a way of working (see Table 4). Four of the teachers had not reflected on why they did not use mathematical modelling in teaching and four teachers related modelling to physics. In addition the teachers were explicitly asked if they thought that modelling is more used in physics or chemistry, and 15 of the 17 teachers agreed (one teacher was not asked, because he only referred modelling to research at universities).

How the teachers work with modelling was also a question, and they described their experiences in working with mathematical modelling in relation to projects, laboratory activities in mathematics and regular classroom activities. Three of the teachers expressed that they were using modelling examples from the book during a ‘project’ in mathematics course D. One of the examples they describe during the D-course project is related to store petrol in the desert, an item used in an investigation by Ärlebäck (2009b). The teachers gave other modelling examples related laboratory activities in mathematics and in regular classrooms activities. Some example originated from physics, such as density in a linear model [teacher C], and working with a pendulum and regression [teacher D]. Others provided examples related to percentage like car price change [teacher M], some related to rental cars or buying mobile phones [teacher O]. One person gave examples of Fermi problems that they work with, like how much sand there is in Skrea Beach and how much water runs under a bridge over the river Ätran [teacher B]. Other ideas of modelling activities were why there are no giants [teacher F] and how many knots are possible to tie on a rope. [teacher K].

Mathematical laboratory activities were often used by four of the teachers, sometimes used by five of the teachers, seldom used by eight of the teachers and never used by one teacher. The main focus of the laboratory activities was related to specific parts of mathematics like geometry, functions, probability and statistics.

The interview question number 6 is related to teachers’ views on assessing mathematical modelling. Twelve of the teachers expressed that they do not or very seldom use modelling examples in their own tests. Reasons for that is time and that their own tests also are used in other classes with other teachers, which the teachers viewed as negative in relation to modelling. Four of the teachers sometimes or often used items with aim to test modelling in their own tests and two of the teachers did not answer the question. The examples given by the teachers were described in general terms like, to construct a model from a text, to set up conditions based on geometrical figures, the cosine theorem, maximum minimum problems (two teachers did not answer this question). The freely available national course tests in mathematics were discussed during the interview and ten out of the 18 teachers expressed a view that modelling items were included in the test and 13 out of 18 stated that they use the national course test as the major factor when they give the students their course grades. The teachers use the national course test mainly for repetition (in 13 cases, five did not answer) and to use some good example to discuss or illustrate on the black board (in four cases).

The last question (nr 7) was about the test items from the “student questionnaire” used in Frejd and Ärlebäck (2011). Figure 2 below displays the teachers’ use similar items in teaching or recognize the items from books.

Figure 2. Frequency of the 18 teachers’ use of test items 1 to 7 in teaching.

[Frequency bar chart showing the frequency of test items 1 to 7 used by the 18 teachers.]

Footnote: Freely available meaning there is no secrecy on the test and it is free to download from the internet.
The height of the piles in the diagram in Figure 2, indicates that items 6 (graphical representation of a model) and 7 (evaluate a model, i.e. a function, based on some conditions) are familiar to most of the teachers. Item 5 (formulate a mathematical condition in an optimization task) is recognised from books or used in teaching by every third teacher. However, items 1 to 4, which are less connected to the mathematical parts of the modelling cycle (see the given example in the method section) seemed not to be represented in many teachers’ classrooms. Arguments for why and why not to use these modelling test items in teaching are described as axial coding categories in the table on the next page.

### Table 5. Results related to interview question nr 7.

<table>
<thead>
<tr>
<th>Why do you use or would like to use the modelling test items in your teaching?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The items have a potential and seem useful</td>
</tr>
<tr>
<td>2. Reasoning</td>
</tr>
<tr>
<td>3. We are already working with some items</td>
</tr>
<tr>
<td>4. To apply mathematics in other places</td>
</tr>
<tr>
<td>5. Relating and connecting to pure math</td>
</tr>
<tr>
<td>6. To enhance students communication</td>
</tr>
<tr>
<td>7. Arguments related to the curriculum</td>
</tr>
<tr>
<td>8. Positive comments to the items</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Why do you not use modelling test items in your teaching?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No mathematics</td>
</tr>
<tr>
<td>2. Problems related the test items</td>
</tr>
<tr>
<td>3. No experience or don’t know</td>
</tr>
<tr>
<td>4. General reasoning</td>
</tr>
<tr>
<td>5. Problems in change of teaching</td>
</tr>
<tr>
<td>6. Other subjects</td>
</tr>
<tr>
<td>7. Not a demand</td>
</tr>
</tbody>
</table>

The axial coding on why do you use these items (see Table 5) has been gathered from 33 open coding categories. The main argument is that the items have a potential and seem useful because they are close to mathematics and might be useful in the mathematics class. There are four categories with 7 teacher answers in each: reasoning (you practise reasoning skills such as making assumptions, reflections, reasonableness), we are already working with some items (this is mathematics, there are similar items in books and national course tests), to apply mathematics in other places (in other subjects and in higher education) and relating and connecting to pure mathematics (graphs is an important tool). Reasons for not using the modelling items were described in the analysis section, but with no result. As seen in Table 5, there are two main axial coding categories for not using the modelling test items. Those are that there is no mathematics in them and that there are problematic issues related to the modelling test items, such as being hard to understand or involving much text.

Finally, the result to the interview question concerning if you ought to work with similar to the test items from Haines et al. (2000) in the mathematics class: 39% of the teachers expressed that you ought to work with all items in the mathematics class, 11% of the teachers expressed that you ought to work with items 5, 6, 7 and one or two extra, 28% of the teachers wanted to work only with items 5, 6, and 7 and 17% of the teachers expressed that is enough to work with one or two of those three items. One of the teachers (6%) did not give an answer to the question because she expressed that she was not sure.

### 6. Discussion

The teachers in this study, most of them experienced teachers, expressed that the major reason for becoming a teacher was positive comments towards the subject of mathematics, which was not unexpected. If you enjoy, like or find mathematics interesting as well as are interesting in teaching (which was the second reason for becoming a teacher) it is understandable that you choose to become a teacher. However, it does not say much about their conceptions about what mathematics really is, which on the other hand is a deep and hard question to be answered in a single question. Some indications of the teachers’ conceptions of mathematics are related to what they express as useful for students to learn. There were ten teachers who stressed the use of mathematics in society as general knowledge useful outside mathematics classrooms. It may indicate that the majority of the teachers have a conception of mathematics as useful to apply in extra-mathematical situations (real world).
addition, the same number of teachers emphasised mathematics as useful in education and science (other subjects, for further studies and as scientific language). One may argue that the use of mathematics in other subjects is also related to apply mathematics in other situations. Scientific/symbolic language, which two teachers expressed, does not necessarily relate to applying mathematics in extra mathematical situations. It might as well be related to pure mathematics like proving theorems in a mathematics class.

The main goal with teaching was a difficult question according to many teachers, and their expressions were focused on teaching mathematical content, motivations for students and students’ understanding of mathematics. In addition a third of the teachers expressed that one main goal was to show the students the use of mathematics in different situations (in extra-mathematical situations, for further studies, in life in general).

Only 50% of the teachers had heard the notion mathematical modelling before taking part in the study of Frejd and Årlebäck (2010, 2011), which may be one reason for some teachers to hesitate while answering the question. A majority of the teachers expressed that mathematical modelling is to describe or simplify ‘something’ with mathematics. The word ‘something’ is related to a course of event, reality, event, connection, something visible and things. The teachers’ conceptions of mathematical modelling may focus on designing mathematical models, where the mathematical models have a descriptive function of something. Also, Frejd (2010) have found definitions in dictionaries and scientific literature of mathematical models related to descriptions of something, were something was described in vague terms like knowledge, as existing system, conceptual system, narrative or reality. The connection to students’ descriptions in Frejd and Årlebäck (2010) about the notions of mathematical modelling is not obvious. The students in Frejd and Årlebäck (2010) stressed problem solving and a method (methods, solving strategies or algorithms) as they main descriptions for modelling. However, three teachers express problem solving as modelling and four teachers express the relation to natural science and especially to physics.

According to 15 out of 17 teachers expressed that modelling is more used in physics or in chemistry. Teacher M gives one suggestion why:

[In physics] where we have another set up, one may think of starting in reality right away and explain something. It is rare that we go directly to physical phenomena without using something from reality and in this way we already have some type of modeling as a prerequisite. It is our tradition one may say, school tradition, which has made it in this way. One has also attempted this in mathematics, but it is often easiest for the teacher in front of the blackboard or involved in a discussion, to start with mathematical concepts instead of modelling.

There are other indications of why mathematical modelling is connected with physics or chemistry, such as the notion mathematical modelling is also mentioned in the in physics curriculum\(^4\). In addition the use of handbooks with physical/chemistry formulas (models) in physics tests\(^5\) as well as the use of the words “designing physical models” in physics textbooks, may connect mathematical modelling to physics (Gottfridsson, Jonasson, & Lindfors, 2003).

The teachers’ conceptions about mathematical modelling were not influenced by any descriptions of the notion in any local curricula, because the notion was not described or defined in any of the local curricula. Reasons could be the vague descriptions in the official curriculum, that the local curricula did not exist or the teachers did not use any local curricula (several teachers expressed that) or the teachers did not focus much about mathematical modelling in the official curricula. The fact that the notion of modelling did not exist in the local curricula may have had an impact on the eight teachers who expressed that they did not work with mathematical modelling in mathematics education, at least to those four teachers who expressed that they had not thought about it. Other main reasons for not using modelling activities are again pointing towards physics, meaning that mathematical modelling takes place in physics education.

On the other hand five teachers expressed explicitly that they used mathematical modelling activities intentionally and their reasons for doing this were a “way of working” and to show how

\(^4\) Retrieved from [http://www.skolverket.se/sh/d/3399](http://www.skolverket.se/sh/d/3399).

mathematics is used outside school. The last reason that modelling activities are useful to show how mathematics is used outside school was expressed by only three teachers, even though a majority of the teachers had the same argument in their goals towards teaching (i.e. the questions What use of mathematics do you see as important for students to learn? What is your main goal with teaching?).

All the 10 teachers who expressed that they used (intentionally and non-intentionally) modelling activities gave examples on such activities. The given examples were originated from different settings from physics, and from more or less realistic problems. However, the teachers only expressed in average between two or three examples of modelling activities. This may also be a consequence of that modelling is not done by all teachers intentionally, or that they did not have a clear conception what modelling is or that they could not remember more examples during the interview.

Two thirds of the teachers did not or very seldom assess mathematical modelling in their own tests, giving lack of time as the main reason for that. No, explicit comments were given from the teachers that they assessed modelling in any other ways. The four teachers who used to assess modelling at least to some extent gave examples related to the mathematical domain. According to Frejd (2011) the national course tests mainly tests the parts of the modelling process related to the mathematical domain. The majority of the teachers in this study expressed that they think the national course test in mathematics test modelling and that the national course test is the most important factor for students’ final grade. This may indicate that the teachers’ conceptions about the national course tests are that they are important and assess modelling. However, their conceptions of modelling seem to be related to parts in the modelling process mainly related to the mathematical domain. Not just because of the way they presented their own test examples or that national course test items stress the mathematical domain (Frejd, 2011), but because the teachers described that they only use or recognize test items from the ‘student questionnaire’ which emphasise the mathematical domain during the interview. In addition, the teachers’ main reason for not using or wanted to use the other test items was that it did not include mathematics. Another reason was that some test items were expressed to be difficult by half of the teachers. It may also be a factor that less than 40% of the teachers expressed that all test items ought to be discussed or used in the mathematics class.

Before making any conclusions one needs to consider that the presented results are based on the grounded theory inspired approach and are only one interpretation out of many possible interpretations of the data. Also, the issue of taking the answers as direct indicators of teachers’ conceptions and not consider adaptive behaviour in the interview situation may be seen as problematic and should be taking under consideration. However, this case study will not claim any general conclusion about teachers’ conceptions about mathematical modelling in Sweden. This study will only provide some indications of teachers’ conception about the notion of mathematical modelling as well as their descriptions of modelling as a part of mathematics/mathematics education.

7. Conclusion and implications

The teachers’ in this study have limited experiences about the notion of mathematical modelling in mathematics education. 50% of the teachers had not heard the notion before taking part in Frejd and Årlebäck’s study (2010, 2011) and no descriptions or definitions were found in any of the teachers’ local curricula. In line with Årlebäck (2010), some teachers expressed that they did not have a clear conception of what the notions means. The teachers’ conceptions relate mainly to designing a mathematical model based on a situation (i.e simplify and describe something with mathematics). One may also conclude that there are no immediate connections on how the teachers’ students described the notion. Frejd and Årlebäck (2010) concluded that students associate modelling with problem solving and as a method (methods, solving strategies or algorithms), whereas the teachers in this study expressed modelling as a activity to design a mathematical model based on a situation, though further research on this issue is needed.

The connection of mathematical modelling to physics or chemistry is a predominant conception expressed by the teachers in this study. 15 out of 17 teachers explicitly expressed that they use mathematical modelling more in physics or chemistry than in mathematics. There were only five teachers who intentionally used modelling activities in mathematics class and the overall lack of assessment in relation to modelling items in teachers own tests may indicate that mathematical
modelling is not a frequently occurring activity in many teachers' mathematics classrooms. Overall, the teachers seem not to give priority to integrate mathematical modelling into their everyday mathematics teaching.

Using a holistic view of modelling according to the definition of modelling competency in Blomhøj and Højgaard Jensen (2003), one may conclude that the teachers' conceptions about mathematical modelling in relation to mathematics (as measured by the test items used in the "student questionnaire") emphasise mainly the mathematical aspects of mathematical modelling. The main argument expressed by the teachers for not using some of the items from Haines et al. (2000) in mathematics classrooms was that those items did not include any mathematics. It appears that Niss' (1993, p. 27) premise "What is not assessed in education becomes invisible or unimportant" illustrates the present situation according to this study. The national course tests stress aspects related to the mathematical parts of the modelling cycle (Frejd, 2011), the teachers have a conception of the tests as being important and the teachers themselves stress the mathematical aspects of modelling.

The statement in the curriculum "The school in its teaching of mathematics should aim to ensure that pupils:… develop their ability to design, fine-tune and use mathematical models, as well as critically assess the conditions, opportunities and limitations of different models" (Skolverket, 2001, pp. 60-61) was given to the teachers in the student questionnaire. It may be one reason for their expressed conception about modelling as a "designing" activity. The other aspects like fine-tune mathematical models and critically assess the conditions, opportunities and limitations of different mathematical models were only expressed in rare cases. When these aspects were not brought up spontaneously by the teachers’ maybe these aspects should be more emphasized in students’ textbooks. Traditional textbooks in mathematics are often used by teachers as a guide for teaching in upper secondary school in Sweden (Skolinspektionen, 2009). The fact that the traditional textbooks do not treat mathematical modelling explicitly (Ärlebäck, 2009b) suggests that the aspects, which were rarely mentioned by the teachers, should be more emphasized in the textbooks.

The teachers in this study expressed that it is important for students to learn that mathematics is useful to apply in extra-mathematical situations (real world). One way to teach students with this goal in mind could be to incorporate more modelling activities in mathematics education. However, as Kaiser (2006) points out, teachers’ conceptions (beliefs) about mathematics is an important reason for the low incorporation of modelling activities in mathematics education. This study also indicates that the teachers’ conceptions about mathematics could be an obstacle for an implementation of modelling activities. 60% of the teachers expressed that you should not work with all test items in “student questionnaire” in mathematics class, because it is not mathematics. Another obstacle or advantage for more implementation of modelling is the indication of mathematical modelling as an activity related to physics or chemistry. There are many students that do not study physics in upper secondary school in Sweden and physics or chemistry problems are only a part of all possible modelling problems. Maybe knowledge from physics or chemistry education about teaching and learning modelling activities can be applied also in mathematics education. Questions that need more research are for instance, what type of modelling activities are done in physics or in chemistry? What aspects of mathematical modelling from physics or chemistry education are useful to incorporate in mathematics classrooms? What aspects are missing?

References


### Interview Guide

**APPENDIX**

**Research Guide**

1. **What conceptions do teachers express about the notion of mathematical modelling?**

2. **To what extent do they believe mathematical modelling activities are a part of mathematics education?**

3. **What conceptions do teachers express about mathematical modelling in relation to national course tests in mathematics, to local curriculum and teaching material such as course literature, laboratory work etc.**

4. **What conceptions do teachers express about the test items from the student questionnaire in Frejd and Ärlebäck (2009)?**

5. **What conceptions do teachers express about teaching mathematical modelling?**

6. **What conceptions do teachers express about mathematical modelling as a didactical tool v.s. teaching mathematical modelling?**

7. **What conceptions do teachers express about different parts of the modelling process?**

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**Research Questions**

A. What conceptions do teachers express about the notion of mathematical modelling?

B. What conceptions do teachers express about mathematics?

C. What conceptions do teachers express about teaching mathematical modelling?

D. What conceptions do teachers express about different parts of the modelling process?

E. What conceptions do teachers express about mathematical modelling in relation to national course tests in mathematics, to local curriculum and teaching material such as course literature, laboratory work etc.

F. What conceptions do teachers express about mathematical modelling as a didactical tool v.s. teaching mathematical modelling?

G. What conceptions do teachers express about the test items from the student questionnaire in Frejd and Ärlebäck (2009)?

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**Auxiliary Questions**

A. What courses and programs do you work with?

B. Why did you become a teacher in mathematics?

C. How long have you been working as a teacher?

D. What use of mathematics do you see as important for students to learn?

E. What is your main goal with teaching?

F. What is an open question? Do you use the national course test as the most important instrument for grading?

G. Could you give some examples from the national course texts about mathematical modelling?

---

**Interview Questions**

A. What courses and programs do you work with?

B. Why did you become a teacher in mathematics?

C. How long have you been working as a teacher?

D. What use of mathematics do you see as important for students to learn?

E. What is your main goal with teaching?

F. What is an open question? Do you use the national course test as the most important instrument for grading?

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**Sub Questions**

- Do you use mathematical modelling in your teaching? If yes, how and how often?
- Do you use modelling items in your test to assess mathematical modelling? Do you experience that national course tests assess modelling?
- What type of items do you use? Laboratory work in mathematics?
- Do you use modelling in your teaching? (If yes) How and how often?
- Do you use modelling in your teaching? (If yes) How and how often?
- Do you use mathematical modelling in your teaching? (If yes) How and how often?
- Do you use mathematical modelling in your teaching? (If yes) How and how often?
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<tr>
<th>Cat.</th>
<th>Open coding to interview Q nr 7. Why not.</th>
<th>A</th>
<th>B</th>
<th>C</th>
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<th>E</th>
<th>F</th>
<th>G</th>
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<tr>
<td>1</td>
<td>Course literature– Is an argument related to students’ literature?</td>
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<td>Students lose pace – Is an argument related to students pace?</td>
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<td>3</td>
<td>Do not know – Is statements like ‘Don’t know’ part of the coding unit?</td>
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<td>4</td>
<td>No mathematics– Is an argument related to no or lack of mathematics?</td>
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<td>5</td>
<td>Common sense – Is an argument related to common sense or general reasoning?</td>
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<td>6</td>
<td>More conditions needed – Is an argument related to that more conditions are needed in order to solve an item?</td>
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<td>No connection to the course –Is an argument that it is not related to the mathematics course?</td>
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<td>No time – Is an argument that no time or more time is needed?</td>
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<td>9</td>
<td>Not for fun – Is an argument related to teaching not just as entertainment?</td>
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<td>10</td>
<td>Difficult to understand –Is an argument related to difficulties for the students in understanding/interpret the test items?</td>
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<td>11</td>
<td>No probability – Is an argument related to a lack of probability in the items?</td>
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<td>12</td>
<td>New type of problem – Is an argument related to new type of items?</td>
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<td>Time killer – Is an argument related to time killers?</td>
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<td>Logic– Is an argument related to logic or logical reasoning?</td>
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<td>Similar experience in real life – Is an argument related to practical experience?</td>
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<td>16</td>
<td>Discussed outside mathematics – Is an argument that some items are discussed in other places than in mathematics class?</td>
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<td>No mathematical thinking - Is an argument related to no math thinking?</td>
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<td>18</td>
<td>Not thought of it– Is an argument related to no considerations before?</td>
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<td>Items are difficult to discuss- Is an argument related to difficulties in discussing a test item?</td>
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<td>Lack of experience– Is an argument related to no similar experience of these items.</td>
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<td>Philosophy – Is an argument related to philosophy?</td>
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<td>Open problems are difficult – Is an argument related to difficulties in open problems?</td>
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<td>No demands from Univ. – Is an argument related to demands from Univ?</td>
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<td>I do not understand – Is an argument related to that the teacher do not understanding of test items?</td>
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<td>Text – Is an argument related to much text?</td>
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<td>Schooling – Is an argument related to the students need work with this continuously?</td>
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<td>No goal –Is an argument related to curriculum goals?</td>
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<td>Extra – Is an argument related to that it is something extra?</td>
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<td>29</td>
<td>Test for higher education – Is an argument related to higher ed. test?</td>
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<td>Physics – Is an argument related that modelling is done in physics?</td>
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<td>Not interesting to students– Is an argument related to a lack of interests of students?</td>
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<td>Nothing to measure- Is an argument related to “there is nothing to measure on in the test items”?</td>
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