The Progressive Continuum of Assessment to Modeling

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“In order to translate a sentence from English into French two things are necessary. First, we must understand thoroughly the English sentence. Second, we must be familiar with the forms of expression peculiar to the French language. The situation is very similar when we attempt to express in mathematical symbols a condition proposed in words. First, we must understand thoroughly the condition. Second, we must be familiar with the forms of mathematical expression.”

George Polyá

Abstract

Often we are so entrenched in the educational systems in which we teach that it is difficult to step outside of our methods and include innovation in the areas of student evaluation and categorization and learning assessment. In this paper, I ask, What methodologies might be appropriate for an examination of the development of mathematical language and how does this relate to second language acquisition? I offer comments on several forms of assessment currently in use in the public school system of the United States, and propose some innovative ideas to alter the ways we use to look at the development of learning.

1. How We Are Categorizing Students in our Public Schools

The categorization of students in the public school system is accomplished by several competing methods: traditional classroom assessment generated by standardized testing and/or individualized student production; referential observation by the child’s instructor; one on one evaluation procedures by ancillary staff members; parent input; and the student’s thoughts on his or her own performance. It is now a common belief that students who test below average in visuo-spatial abilities will have less success in mathematical skills acquisition (Zera & Lucian, 2001). Members of the predicted “less successful” populations which test lower in visuo-spatial abilities are often overwhelmingly pinpointed in minority student groups (Cummins, 1989).

The link between visuo-spatial abilities and mathematical skills is historically entrenched, particularly when it comes to group assessment of children in public school settings, but recent experimental evidence shows that this link is more complex than a simple correlation between an indexing and categorization processing expertise and skill presentation through practice (Pon-Barry et al., 2006). The acquisition of what Dehaene (1997) calls a “number sense” incorporates and interfaces skill sets from different parts of the brain, some which are also activated in language acquisition, and some which are activated solely during numerical calculation operations. Even though this body of experimental evidence continues to accumulate in larger and larger quantities, our standardized testing rubric remains stagnant in its approach to assessment: with the evidence that various skills and abilities have a more global application to learning, one might expect that the interpretations of standardized tests be reconfigured to reflect the wider range of abilities which may be emerging from the testing process.

I want to suggest a continuum which represents the movement from numeracy, numerical literacy, to literacy and back, and that the policy of testing in the public schools needs to take this continuum into account as testing becomes a more and more integral part of the daily routine of the U.S. public school student (Dwyer, 1998; Romberg, 2001). Numeracy is a term, which is gaining in use in the United States, after two decades of work in the United Kingdom and Australia (Matang & Owens, 2004). It refers to the quantitative literacy, which develops within a person over the course of her education and her life, and the Numeracy Working Group from Australia (ILSS, no longer available) came up with the following definition:
Numerate behaviour is observed when people manage a situation or solve a problem in a real context; it involves responding to information about mathematical ideas that may be represented in a range of ways; it requires the activation of a range of enabling knowledge, behaviours, and processes.

Within the testing framework in the public schools and regarding the students who enter school with limited English proficiency (LEP), primary interest has focused on their verbal abilities. Now there is evidence that the second language is acquired differently than the first (Mahon & Crutchley, 2006), and there are also studies that show the distinction between the acquisition of language and the acquisition of mathematical reasoning (Brannon, 2002, 2005; Feigenson et al., 2002; Varley et al., 2005). If this mounting evidence holds to be true, the testing of verbal abilities might be shown to be inadequate for assessing both language and mathematical skills in LEP students, and yet the emphasis on testing students in verbal areas to determine learning disabilities continues. In the future, we may have to look to a more broad-based yet individual assessment framework to find out how our children are learning, and how best to help them acquire the skills they’ll need to be part of their world.

Currently, assessment in the United States often assumes the form of specter, or a monster out of control, and the issues concerning standardized testing seem to occupy most of the “assessment” discussion. The term evaluation, frequently used in the context of Special Education, sometimes carries with it a negative implication of locating the problem within the child, and not within the system (Cummins, 1989). The result of evaluation is often placement in a special program, sometimes even in a special school, and includes the design of an individual education plan (IEP) for the child. For the purposes of this paper, we will not include evaluation for special education services, and we will only discuss the forms of assessment of children to determine mathematical skill levels and to predict future mathematical success.

1.1 Forms of Assessment

Traditional assessment, or summative assessment, “takes place after a period of instruction and requires making a judgment about the learning that has occurred” (Boston, C., 2002, p. 1). There are many problems that are identified with these norm-referenced tests (NRT), and these problems have been known for over fifteen years (Frechtling, 1991). Some of the problems with NRTs that have been proposed are:

1. NRTs measure a student’s behavior relative to his or her peers, not to established criteria of knowledge or behavior.
2. The multiple-choice format of NRTs corrals the items into concrete questions, covering lower cognitive levels.
3. When used to ensure accountability, the NRT format limits, and may even drive curriculum.
4. NRTs tend to be culturally and linguistically inequitable (Stevens, 2000, p. 51).

This final fourth point is where the concerns of this author lie; with the current influx of public school students from countries outside of the United States, the test score gap between English language learners and other students grows larger with each passing year (Bielenberg & Wong Fillmore, 2004-2005).

1.1.1 Alternative Assessment

Some new methodologies of assessment that might be helpful come from the current research into culture-fair assessment (Verney et al., 2005; Fagan, 1992, 2000; Fagan & Haiken-Vasen, 1997) and formative assessment (Boston, 2002; Wolfendale, 2004). Culture-fair assessments include information processing and psychophysiological assessments to “reduce cultural biases in standardized assessment” (Verney et al., p. 316), and this topic will be combined with the notion of dynamic assessment, coined by Lev Vygotsky (1931).
1.1.1.1 Formative Assessment

In the interest of evaluating the evaluation process, there is a field called formative assessment (Black & Wiliam, 1998), which helps all of the parties involved in the assessment, evaluation, and instruction process to observe themselves and each other as the process evolves. Formative assessment is “the diagnostic use of assessment to provide feedback to teachers and students over the course of instruction…teachers assess how students are learning and then use this information to make beneficial changes in instruction” (Boston, p. 1). Can assessment be continuous without interfering in the developmental process or becoming invasive in the child’s and in the family’s lives? And can this assessment process become more inclusive of children’s psychological and emotional development, so that the child, the teacher, and the child’s family learn how to observe the ongoing processes of growth and development? To begin to answer these questions, we can look at Black and Wiliam’s (1998) work in the area of formative assessment, which includes:

− Teacher observation, classroom discussion, analysis of student work;
− Adjustment to instructional strategies, reteaching, opportunities for practice of skills;
− Feedback which focuses on improvement as a result of effort, counteracting the cycle of blaming poor performance on lack of ability;
− Learners also learn to evaluate by self-monitoring.

Formative assessment is ongoing in the classroom, so it becomes part of the child’s school day. In this way, assessment becomes normalized, and the child begins to see it as part of a daily routine. If we extend the framework for formative assessment into the areas of self-monitoring of a child’s acquisition of mathematical skills, how much earlier might potential problems be identified?

Some of the strategies designed for the formative assessment of students’ understanding might be adapted by teachers for use in determining social and psychological development. For instance, instructional units on number sense development can be adapted for use even with the youngest of students. Talking about how numbers play a part in daily life is often part of the school day, so why not use it as part of the curriculum? Here are some of the ways for teachers to encourage a classroom environment of understanding about the topic of numbers:

− Invite students to discuss their thinking about a question or topic in pairs or small groups, then ask a representative to share the thinking with the larger group (sometimes called think-pair-share);
− Present several possible answers to a question, then ask students to vote on them;
− Interview students individually or in groups about their thinking as they solve problems;
− Ask students to summarize the main ideas they’ve taken away from a discussion.

The concepts of formative assessment in the areas of educational and psychological development need to be included as part of the professional training for teachers, for professionals working with children, and for parents. Once these concepts are learned by parents and professionals, the use of them will come to pass only if they receive the proper support from their schools and communities. Matanga and Owens (2004), in discussing the implementation of mathematics curricular changes, emphasize three points in their quest for dramatic change in the curriculum:

1. Teacher beliefs and values must align with the rationale for curricular changes;
2. Proper training programs and subsequent support must be in place to allow teachers to do an in-depth investigative study of mathematics content knowledge (including discovering the cultural context of the students);
3. They (both parents and teachers) need to become aware of their new role – they change from being an authority and transmitter of mathematical knowledge to a facilitator of the teaching-learning process.
1.1.1.2 Dynamic Assessment

Because this third point connects closely with our final topic, we can now turn to the concept of dynamic assessment, which emerged from the work of Lev Vygotsky, a Russian scholar who developed psychological theories following the onset of the Russian Revolution. This revolution and its ensuing disorder orphaned hundreds of thousands of children who then lived on the streets of the cities, and it has been proposed that Vygotsky began his work on assessing children who had emotional and intellectual problems as a response to the plight of these orphans (Metheny, 2003). As Vygotsky envisioned the assessment of children who had experienced disruptions during development, his theory of dynamic assessment presents a framework within which the child’s environmental influences are gauged by comparison of his or her current level of development with the future level of development for the child, their potential development and where they are in the process of realizing this potential (Poehner & Lantolf, 2005). If the true tenets for dynamic assessment are followed, this form of assessment is “tuned to the abilities that are maturing,” and this tuning “is continually renegotiated” (p. 29). Dynamic assessment is inseparable from the instruction; “they form a unity necessary for learner development” (p. 29). Similar to a dance choreographed to music, the interaction between learner and teacher ebb and flow as assessment and instruction intertwine. Poehner and Lantolf (2005) compare dynamic assessment to a perspective in which assessment and instruction “are seen as two sides of the same coin…true assessment is not possible unless it entails instruction, and vice-versa” (p. 30).

1.1.1.3 Culture-fair Assessment

The two sides of a coin can also refer to the last topic we will cover, the culture-fair assessment work of Stephen Verney of the University of New Mexico. In his doctoral work with diverse populations, Verney looked at underlying biases in psychometric assessment, including intelligence testing, which typically includes visuo-spatial capacity measurement and thus, predictions of mathematical performance. Because the assessment of mathematical abilities in diverse populations often includes a look at the language of mathematics and how it differs from culture to culture (Echevarria & Graves, 1998; Krause, 2000; Perkins & Flores, 2002; Ron, 1999; Tevebaugh, 1998; Torres-Velasquez & Lobo, 2005), the development of additional measures to show the learning capacities of these diverse populations is essential. Verney et al. (2005) shows that the ethnic backgrounds of diverse cultures do not affect their learning abilities, as so many of the standardized tests have suggested (MacMillan, Gresham, & Siperstein, 1993); he uses psychophysical measures (pupil response data to light) as proof that this

...unique information about an individual’s cognitive processing can be obtained by recording psychophysical measures during cognitive task performances while simultaneously gathering more traditional behavioral response (e.g., correct response, reaction time) (p. 305) data.

It’s work such as this, combined with measures that can be used to detect possible stages of student confusion during instruction, that will pave the way for reform in the assessment arena.

1.2 Assessment and Pedagogy in Second Language Mathematics Learners

The need for alternative assessment procedures is one of the issues indicated by this increase in the gap between English Language Learners and their counterparts in our schools. A suggested parallel practice is the use of ‘diagnostic teaching,’ a concept that helps to identify children’s strengths and weaknesses, along with assessment; ‘diagnostic teaching’ is designed to help teachers adjust their teaching styles to the needed instructional areas (Misaïlidou & Williams, 2003). Diagnostic teaching involves the exposition of difficulties that the student is encountering by drawing attention to the areas of possible confusion, thus clarifying the essential characteristics of the problems that the students are given to solve (Bell, 1993, p. 27). In mathematics instruction, the presence of misconceptions often causes problems for the instructors and the learners; in multicultural classrooms, the instructors strive to facilitate mathematics
understanding, and for this they must be bilingual in the register, the vocabulary, of mathematics in both languages (Ron, 1999). Because language acquisition is part of the learner’s cognitive skill development, in a learning environment “...cultural differences emerge against a backdrop of universal skeletal principles of conceptual development” (Medin et al., 2002, p. 10). For the purposes of assessing the linguistic development of mathematical language, we should consider that the basic process of language acquisition is tied to the ability to acquire then employ morphological and even syntactic patterns in the course of language use (Givon & Malle, 2002). This ability to predict the pattern of and employ a form while acquiring and using new lexical items in previously acquired patterns may also be linked to the ability to estimate either the solution to a mathematical problem or to guess at how a solution should be arrived at (Dehaene, 2008), which further connects the language acquisition process through the developing skill of categorization. An instructor who uses diagnostic teaching as a format for exposing comprehension difficulties among her students can therefore look at language use to help her determine where the confusion might be occurring. But it also requires a deeper awareness of the culture-specific usage of mathematical language.

Data that emerge from studies of the multicultural representation of mathematical concepts contain hard evidence that although there may be a difference in the way cultures describe mathematics linguistically (Levinson, 2003), this difference does not extend to the ability of the people of that culture to perform mathematical calculations (Zaslavsky, 1996). In linguistics, we might say that viewing mathematics through an alternative cultural lens means that we are employing a schema: a schema is sometimes defined as a situational environment that contains its own set of matching concepts. Schemas have also been described as "cognitive constructs which allow for the organization of information in long-term memory (Singhal, 1998; Widdowson, 1983), and Cook (1989) states, "the mind, stimulated by key words or phrases in the text or by the context, activates a knowledge schema" (Cook, 1989, p. 69). For example, there is a “restaurant schema” in which conceptual ideas of -- waiters, menus, tables and chairs, maybe even, candlelit tables, and those sorts of things -- occupy that space typically evoked when the word 'restaurant' occurs in language use; further, when someone talks about ordering food and leaving a tip, we generally know that they are conversing about a restaurant experience. Linguistic schemas and their accompanying experiential realms often transcend language boundaries, meaning, there are similar concepts in many different languages. But, since schemas also alter from environment to environment and not only from culture to culture, just because someone is bilingual does not necessarily mean that they understand the schemas that are present in both their first and their second language.

Sometimes elementary level bilingual teachers do not always know the specialized language in every content area they teach, notably in mathematics. The everyday language of mathematics, which includes everyday experiences with using mathematical processes, needs to be used in both languages (or all languages) in order for the students to understand the examples offered. The students need to be involved in producing the language of math in the classroom, and there should also be opportunities for the students to share the different ways they have of expressing mathematical concepts in their first language; often this leads to a deeper understanding of the concepts for all of the students (Perkins & Flores, 2002).

The teachers can ensure that the children are exposed to multiple ways of expressing the same mathematical concept whenever possible. A teacher might, for example, outline a problem one way then also explain it in a more graphic way (Ron, P. 1999). Students may have varying levels of cognitive ability, but this does not prevent them from acquiring some level of steady competence in mathematics (Echevarria & Graves, 1998). The language of mathematics, just like the language of shopping or the language of a religious interaction, is not universal just because the instructional wording is translated from one culture’s language to another. People have different ways of using math that relate to their own unique everyday cultural or ethnic experiences (Tevebaugh, 1998) and different students learn mathematics in different ways.

“For learning to occur...,” says, Jane Watson, “students must feel some dissatisfaction with current ideas and the new ones must be intelligible and appear plausible” (2002, p. 1). The presentation of new information is bound to exert additional stress on the learner’s ability to process new knowledge, but
the careful and consistent introduction of new concepts can have a positive effect on conceptual change, and the assessment of the learner’s processing and integrating efforts can be assured with careful collaborative efforts involving all the participants in a learner’s multiple environments. Sheila Wolfendale (2004, 2005) has written extensively on a triangular partnership model of assessment for use with professionals, students, and with parents (or caregivers). She believes that “an ethical assessment code of practice could ensure that the rights of all involved are respected and exercised” (2004, p.6). In fact, the activity of introducing new information which conflicts with one’s existing conceptual framework (and, in turn, the assessment of this activity) might provide a basis for a student to begin to experience dissatisfaction and turn to seeking out alternative means of making sense of a problem and solving it in other ways.

1.2.1 Conflict as Learning Motivation

The discomfort that accompanies the cognitive conflict that occurs during learning attempts can be used to instigate learning; the motivating power of conflict or paradox in the course of mathematics instruction is well documented (Shaughnessy, 1977; Movshovits-Hadar & Hadass, 1990; Wilensky, 1995; Lesser, 1998). This motivation may be accounted for by considering the selective attention brought to bear in “Slobin’s (1996, 2003) thinking-for-speaking hypothesis, which states that linguistic influences occur when language is used during a task. The idea is that, in speaking, we are induced by the language we use to attend to certain aspects of the world while disregarding or de-emphasizing others” (Feist & Gentner, 2007, p. 283). So, the attention that we are directing at the task which is occupying us is influenced not only by the language which is used in the course of the task, but also by the language used in possible earlier experiences of similar tasks. Because so much of mathematics is new information (whether the instruction is in one’s first or second language), the stage where the new information is compared to the already present set of information is often more of a contrasting than a comparing stage, presenting a conflict between the existing and the new information, but a learner can reorient herself by employing an awareness of the process which is taking place. If these languages differ, the conflicting information may be too challenging for learners to resolve, and yet, if a learner is able to gain an awareness of the conflict as it occurs, the voluntary attention which is activated during the conscious awareness of learning of higher level concepts will be able to override the conflict and use it to provoke the integration of a novel concept.

The more natural human process of integrating new information – of learning – would seem to provide a description for the possible continuous acceptance of alternate explanations to be inserted and tried out in the learner’s categorical/conceptual framework; however, the unusual resistance of the learner’s self and co-constructed ‘naïve’ views of the world (Christou et al., 2007) is problematic. The stabilization of a learner’s conceptual framework, of her preconceived (well-conceived) notions of how her world operates are very resistant to the introduction of novel concepts; Vosniadou (2008) refers to a child’s conception of the world and how it works as ‘naïve physics,’ reinforcing her self-initiated explanations of the forces at work in her world. Because young learners organize information into an approximately coherent (for them) framework, the new information presented as input to learners does conflict with their self- and co-constructed frameworks. But rather than assume that the learner continues to believe her ‘naïve’ world view – might it not also be possible that the alternative explanations presented in science and mathematics classes are simply so divergent in their form of presentation from the learner’s more natural process of knowledge acquisition of her ‘naïve’ framework that the newer approach or information doesn’t stand a chance when pitted against the entrenched, recursive network of existing and self-and co-constructed explanations?

Tall mentions the issue of conflict in the mind of the mathematics learner as early as 1977, suggesting that the teachers look for “confusion, annoyance, fear, or just a dull lost look in the eyes” (p. 11) to help them realize the occurrence of conflict during instruction and redirect or resolve it, and methodologies which attempt to control the accommodation of new ideas in a classroom setting in order to formulate a way to encourage clear and productive discussions to allow self-correction of “unsatisfying”
(and probably mistaken) concepts are now in place in current mathematics curricula (Watson, 2007). But it might also be possible for learners to embrace, then to make use of their own conflict.

1.2.2 Self-reflection as a Way to Assess Conflict during Learning

Because learning mathematics is comparable to building a structure step-by-step (similar to telling a story), each step can be checked and evaluated (Lin, Yang & Chen, 2004) before the next step is put into place; the main point of this possibility is its broad application: even if the step-by-step process differs from learner to learner, the process of an extra self-checking step will still be valid because it is accomplished by means of self-reflection. Self-reflection is a way to evaluate: one’s present stance in a knowledge base; the content of a knowledge category as it stands; and the organization of its constituting members. Educators can encourage learners into a self-awareness of the way they conceptualize, and this awareness of how one represents one’s own thinking may help a learner to locate conflicting thinking patterns, which, particularly in mathematical skill acquisition, have been previously been constructed in order to accomplish problem solving. If these conceptual methods of problem solving are constructed by the learner before formal mathematical learning takes place, say, in the situation of learning how to do a geometric proof, the persistence of these “home-made” methods for solving problems can create difficulties for the learner when she attempts to integrate a formal problem solving framework (Yang & Lin, 2008).

This awareness is cultivated through the ability to question; “…it is important for teachers to challenge [learners]…by asking why they think a particular result is true” (Christou et al., 2004, p. 217). Utilizing the incorporation of controversial topics in mathematics lessons can be very beneficial for the learners: it inspires a “greater mastery and retention of subject matter, higher quality decisions and solutions to complex problems and more frequent creative insights” (Lesser & Blake, 2007, p. 5). Not only do these methods allow the learner to see that the answers in mathematics are fallible, but it allows their cognitive framework to accept that the integration of conflicting novel concepts is a more naturally occurring process. The learner becomes self-reflective in her learning, and in the course of practicing the metacognitive act of thinking about how she is thinking, she is able to “take control of [her]…own learning by defining goals and monitoring the progress toward [her]…achievement” (Katz, Sutherland, & Earl, 2005, p. 4). The issue of self-monitoring enters when students learn what to look for in themselves as they develop. Drawing, writing, and sharing in groups are ways for students to become aware of how they are feeling about the topics they cover during the school day (Evans and Reilly, 1996). Even portfolios, or collections of student work, can be used formatively, and the development of problem solving with mathematical questions can be included in this portfolio. With careful annotation of the entries by either the teachers or the students, development over time can be observed (Duschl & Gitomer, 1997); instructors can use interactive discussion or self-mediated reasoning to realign the characteristics of the membership of a category, in the process, re-determining its constituent membership.

In order to help students to discover the conflict between their own long-held ‘naive’ mathematical beliefs and newly presented material, a method of analysis that allows the student to take account of her own thought process was developed by Laburu and Niaz (2002); in the course of their study, a student recorded his thought process throughout introductory work with a new concept, and the simple fact that this student could refer to his own thought process as he worked through his own ‘hard-core’ concepts then gradually began to integrate new concepts enhanced the student’s ability to reform his basic beliefs. This methodology “…provided a glimpse of how a particular student grappled with conflicts in order to facilitate progressive transition in understanding” (p. 211); and the record of the student’s problem solving attempts throughout the new concept acquisition process allowed the student to self-reflect. Pursuing the analysis of the language used by learners in the course of their acquisition of mathematical skills can help to uncover the relationship between a learner’s ‘hard-core’ beliefs, which students are often unwilling to question, and the introduction of new material, which contains the potential to alter those beliefs and enhance skill development. And when the learners themselves are analysts of their own problem solving methods, this gives them a powerful tool to begin to explore their own conceptual framework. But, how
can we refine this process of self-reflection into a framework that can be used by learners at multiple levels of education?

1.3 Modeling the Self-Reflective Process

In order to implement a framework of self-reflection into the process of learning new concepts, I’d like to introduce the modeling perspective on learning; modeling is a cyclic process which contains the following steps:

− A problem situation is interpreted;
− Initial ideas (initial models…) for solving the problem are brought to bear;
− A promising idea is selected and expressed in a testable form;
− The idea is tested and information from the test is analyzed and used to revise (or reject) the idea
− The revised (or a new) idea is expressed in testable form; etc.

(Zawojewski et al., 2008, p. 6).

Additionally, “a modeling perspective on learning is based on the assumption that students do have relevant ideas to bring to bear on most problem situations” (p. 6), and by extension, this perspective would enhance the ability of students who are learning in a second language to contribute their own beliefs to the learning process. In the course of modeling, the self-assessment that is required “…helps students develop their capability and comfort with approximations and estimates that are often needed in the early stages of…problem solving” (p. 6).

Modeling might be seen as a self-reflective self-assessment – a learner’s construction or representation of how she is understanding or working at understanding a problem helps everyone involved: the learner records an internally visualized process for her own benefit and for the benefit of co-learners; she is also showing her level of skill acquisition to her instructor. Another advantage of modeling one’s own mathematical thinking is the revelation of misconceptions through language usage. The second language learner can look at the record of her problem solving processes in multiple ways:

1. Through an examination of the language she is using to describe her thinking, she can compare her own usage with that of the instructor or of the text to check to see if she is assimilating the information;
2. She can work with her classmates in a group to locate patterns of similarity in both their own ‘hard-core’ beliefs and their combined description and gradual integration of the newer concepts;
3. The record of a student’s problem solving process can be both oral and written: the opportunity to record oneself in the process of explaining, listen to the recording, then reading a transcript of that process as described will allow further exploration of the student’s reasoning process.

In order to accommodate varying levels of student mathematical skill acquisition, the transcripts need not be done by the student (although older students would certainly be capable of transcribing their own oral records); having and being able to refer to an ongoing record of how her thought processes are altering throughout instruction offers insight into the learning process itself, which can benefit the student not only in mathematics, but across disciplines. The transcript, the product of the oral record, might be viewed as a representation of the learner’s conceptual processes – a sort of model of what she brings with her when encountering the new concept, how she manipulates her current beliefs as she attempts to integrate the new information into her cognitive framework, and how she attempts to use the newly integrated information as she addresses a problem.

In his work, Vygotsky (1978) discusses a methodology called, in English, “double stimulation”; he uses this method, for example, in order “to trace the development of arithmetic skills in young children by making them manipulate objects and apply methods either suggested to them or ‘invented’ by them…” (pp. 74-75). If we relate this concept of “double stimulation” to the self-reflective method discussed above, we can conclude that the learner, in the course of orally recording her problem solving processes prior to, during, and after she has received instruction introducing a new concept offers her the possibility for self-exploration into her previous and potential beliefs through the method of recording. In a
description of the method, Sakharov lays out the procedure of “double stimulation”:

…the principle of the experiment is that the series of objects is given to the child immediately as a whole, but the series of words is given gradually, and the nature of the double stimulation continually varies. After each such change we obtain the child’s free response, which enables us to assess the changes that have taken place in the child’s psychological operations as a consequence of the fact that the series of objects now contains a new element from the verbal series. This enables us to assess the degree to which the child makes use of words. Of course, the task can be accomplished correctly only if the experimental concepts that underlie the test words have been formed.

(Sakharov, 1930, p. 32)

Although the method of self-reflection differs in many ways from the psychological experiment using “double stimulation,” it is possible to draw several parallels. First of all, the step where a child receives the entire series of objects might be compared to the set of beliefs about, for example, arithmetical operations, that a student might bring to the activity of learning geometric concepts. Secondly, the introduction of the new concepts containing the framework of geometric operations, is similar to the gradual introduction of the usage of the words which correlate with the “objects” (the previously held beliefs about arithmetical operations). Looking for free responses with regard to the introduction of an geometric concept could be associated with the learner’s recorded descriptions of her thinking processes as she acquires and attempts to use these new concepts.

This student-kept record, similar to a dialogue journal that is used in ESL and in language arts courses (Mahn, 1997), also lends itself to further analysis by the instructor and even by more detached analysts: researchers, administrators, evaluators, and parents, who will be able to view the record through their theoretical frameworks, organized evaluation rubrics, or personal interest. In particular, the linguistic analysis of the mathematical language used in the course of learner acquisition of mathematical skills might help us to focus on the conceptual reorganization that is taking place during this difficult but important process.

1.4 Incorporating Linguistic Analysis into Assessment

Cognitive linguistic analysis can be used to explore the linguistic representation of human conceptualization processes in order to reveal the connections and interactions of thinking and speaking. In both linguistic analysis and in academic assessment, we are attempting to gain insight into human conceptualization and its manifestation in the course of communication; as a basis for this investigation, let us agree that “…conceptual space is universal, although it may be influenced by linguistic conventions…” (Croft, 2001, p. 2), which offers advantages when we examine human speech as a potential output of conceptualization processes, since our idiosyncratic usage of conceptual space is undoubtedly diverse, and yet, human conceptual space is probably the same (Janda, 2005).

In the course of the task of orally recording one’s thinking processes, a body of discourse will be generated which opens itself to many layers of analysis. There is every chance that this task will create “‘metacognitive,’ or reflective, opportunities [that] can help individuals take control of their own learning by defining goals and monitoring the progress toward their achievement” (Katz et al., 2005, p. 2326). The multiple levels where analysis might take place have the potential to yield insight to all of the participants in the learning process, including: the learner; the learner’s peers; the instructors; the administrators; the evaluators; and the researchers. Although each participant looks at the product of the oral record in a different way, from the rudimentary self-reflection of the learner to the exacting and often complex linguistic analysis, which can take the form, in this case, of discourse analysis.

Discourse analysis has as its premise an emphasis on the importance that it places on the examination of critical interrelations between utterances (Grenoble, 2004); it also insists that natural language that is contextually situated and not always “strictly linguistic, but social and/or cultural” (Grenoble, p. 3), be used in the analysis. Although the less technical analysis of classroom discourse encountered in the transcribed records of the learning process of mathematical skills does offer very real
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insight to the learners and to their peers and instructors, there have also been attempts to perform a more elaborate linguistic analysis on these records of individual learners’ mathematical skill acquisition.

The development of the software program, Pepite, in France in 2004 had as one of its goals the analysis of “…the language created by students that combines mathematical language with natural language…[hoping that it would] demonstrate an early comprehension of mathematical notions” (Normand-Assadi, 2004, p. 381). The software does record the language used by students when they try to explain how an algebraic procedure is accomplished, but the analysis has not been successful in assessing the “correctness of justification in ‘mathural’ [the language of math + natural language] language” (p. 388). The authors suggest a categorization framework that would allow the diagnosis of incorrect usage, but it’s possible that a broader, discourse analysis would at least lend some insight into what is going on conceptually with the learners’ attempts.

Since the focus of the analysis is the oral record and its transcriptions, varied theoretical frameworks might be applied for a linguistic analysis of the results. Two such frameworks are the Relevance Theory model and the Transactional Discourse Model, in these models, an examination of “shared knowledge sets and an intersection of common concerns…[are] based on the premise that the explanation of many discourse phenomena (such as word order) can only be found with reference to the psychological states of the interlocutors” (Grenoble, 2004, p. 23). Frameworks such as these would prove helpful in investigating the discourse record to define the “belief systems, [which] are one’s mathematical world view…One’s beliefs about mathematics can determine how one chooses to approach a problem, which techniques will be used or avoided, how long and how hard one will work on it, and so on” (Shoenfeld, 1985, p. 45).

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The study of intonation, in which computer software is used to measure the sounds waves of the individual units and constructions in discourse, is also useful in this type of analysis (Grenoble, 2004, p. 24). “Intonation has been studied from two essentially different views: the acoustic approach measures intonation units in terms of changes in fundamental frequency (F_0) while the perceptual approach relies on auditory perception, and intonation can be defined in terms of pitch” (p. 24). A fourth framework useful in discourse analysis is that of information structure; “information structure examines how information is ‘packaged,’ or linguistically encoded, and why one or another structure might be selected to convey a given chunk of propositional knowledge” (p. 25).

…much of information structure is territory shared with cognitive linguistics and cognitive science. In fact, the two disciplines (cognitive science and discourse analysis [, the areas which this paper seeks to invoke in its attempted investigation into how best to observe the acquisition and subsequent expression of mathematical language] ) may approach the same issues in language data but from different angles, and the results from each approach inform the other (Grenoble, 2004, p. 26).

We need to remind ourselves that these methods of analyzing the products that learners can offer by making oral recordings of their problem solving processes are methods that are useful to learners at all levels; this type of analysis also cuts across cultural boundaries in its applications: it offers a way to explore the interaction of more than one language structure when a learner is attempting to solve mathematical problems.

In this respect, learners will reveal their conceptual processes for analysis, whether they are using their first language or are attempting to learn in a second language. In the body of research on student beliefs in the area of mathematical skill acquisition, “a coherent theoretical framework that identifies the different categories of students’ beliefs in relation to each other is lacking” (Op’Eynde et al., 2006, p. 86). Because of its alternative analytic approaches, cognitive discourse analysis may be able to fill this void.

2. Conclusion

Two new methods of assessment for categorization of students are offered in this paper. First of all, the cognitive linguistic analysis suggested in this paper adds an interdisciplinary dimension to the investigation of individually and contextually based analyses of student belief systems and their effects
upon the competent acquisition of mathematical language and the implementation of the skills involving this language. This paper also suggests that focusing on the normalization of conflict that might be achieved through a growing self-awareness of how one’s own thinking and learning processes function would be helpful to learners acquiring mathematical concepts. Reconciling the retention of long-held beliefs while opening up to alternative views may indeed be part of the process of cognitive maturity, but the early encouragement of younger students to engage in creative attempts to observe their own learning processes may help to accustom them to the frequent encounter of new concepts in academic areas. Where an instructor may not succeed, or may succeed very slowly, in altering a ‘hard core’ belief, the learner herself may have better results in intentionally, volitionally, replacing a long held belief with a novel one, given the opportunity to discover on her own that the old belief does not yield the correct solution and that alternative approaches work better. Recall from our earlier discussion that the process of language acquisition involves the incorporation of new concepts (as words or groups of words) into one’s mental lexicon, where the frequent introduction of new information has no doubt accustomed the learner’s mind to almost constant conflict (in the form of these newly encountered and experienced concepts), leading to a willingness to pursue multiple paths to a solution and an acceptance that sometimes being in error is merely part of learning.

Secondly, the activities of the mathematical modeling of problems and the oral record of the learner’s attempts while engaging in the “activities involved in the process can lead the child to understand a situation or context and get to know the mathematical language that permits him or her to describe, represent and solve a real-life situation or context and to interpret/validate the result within this same context” (Biembengut, 2007, p. 452). The methods described here would be of use most importantly to the learner herself; as a process of self-reflection, these methods offer insight into the way in which “[c]ognition exploits repeated interaction with the environment, not only using the world as its own best model, but creating structures which advance and simplify cognitive tasks” (Anderson, M.L., 2003, p. 126). The successful implementation of such an innovative method depends primarily on the teachers; “[a] key issue in fostering such innovations is teachers’ commitment to understanding students and their cognitive processes as well as the curricula they teach” (Dwyer, 1998, p. 138).

The preponderance of profound problems in our world concerns all of us, and the solutions to these issues require that we have and make use of all of our intellectual resources. Realistically, the limitations we place on our children, in particular on our second language learners, by assessing them through the use of inappropriate measures, limits all of our opportunities; the child who is discouraged from pursuing advanced coursework in a content area such as mathematics or from attending college may have become the adult we needed to redirect a destructive governmental policy or to avert a planetary disaster. Our children are our future, and we cannot afford to lose this valuable resource; we must work very hard to offer every child a chance to learn and to contribute to our world and its ways.

References


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