Three teaching principles for fostering students’ thinking about modelling:
An experimental teaching program for 9th grade students in Japan

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Abstract
This study reports on an experimental teaching program to foster students’ thinking to promote modelling which consisted of nine lessons for Japanese 9th grade students. In this program, the following three teaching principles were emphasized: (TP1) Conflicting Situations: The teacher presents meaningful conflicting situations arising from particular modelling problem so that students can derive key ideas from these situations, (TP2) Repeated Connections: the teacher makes repeated connections between students’ thinking that promotes modelling and (TP3) Spiral Reflections: The teacher encourages students to reflect on their thinking throughout the series of nine lessons. As a pre- and post-test, the (PISA) Heartbeat problem and a general question on modelling were used before and after the teaching program to examine what transfigurations or shifts in students’ knowledge of modelling can be observed. This general question had the following sub-questions: What processes are required to build up a functional model? What ideas are important for developing an effective functional model? A graduated coding analysis was applied to assess students’ responses to the Specific (Heartbeat) question and to the General question. A graduated coding analysis was applied to assess students’ responses to the Specific (Heartbeat) question and to the General question. Significant gains in students’ post-test codes/scores on both questions were obtained. This and the resulting discussion provide strong evidence that a teaching program based on the above three teaching principles is substantially effective in fostering students’ thinking to promote modelling.

Keywords: mathematical modelling, teaching program, teaching principles

1. Introduction

Our previous paper (Ikeda, Stephens and Matsuzaki, 2007), reported on an intensive teaching program for high school students by treating multi-choice modelling tasks. These were shown to be effective especially through teaching methods using meaningful conflicting situations for students. These were also successful in eliciting students’ different opinions so that students can derive the key ideas of modelling from it. This study reports on a longer term teaching program for junior high school students retaining this teaching principle and adding other principles that will be shown later.

The objective of this study is to examine what transfigurations or shifts in students’ knowledge of modelling can be observed as a result of a series of experimental teaching focused on students’ thinking that promote modelling in Japanese junior high school.

2. Methodology

The design of this study is composed of the following: (1) A set of three teaching principles discussed below, (2) A planned program of experimental teaching, and (3) A set of assessments to evaluate the effectiveness of the program.
2.1 Three teaching principles emphasized in a series of teaching

We set the experimental teaching composed of nine lessons for Japanese 9th grade students. In a series of teaching, the following three teaching principles are emphasized.

*(TP1) Conflicting Situations:* The teacher presents meaningful conflicting situations arising from particular modelling problem so that students can derive key ideas from these situations. After presenting the problem, the teacher elicits a variety of ideas from students which are intended to resolve the conflicting issues they have identified. Students share ideas and realize the process by which conflicting understandings may be resolved.

*(TP2) Repeated Connections:* The teacher makes repeated connections between students’ thinking that promotes modelling. Especially important throughout the series of lessons is the idea of setting up assumptions in modelling. The teacher reminds students repeatedly of the need to clarify vague points and to set up clear assumptions.

*(TP3) Spiral Reflections:* The teacher encourages students to reflect on their thinking throughout the series of nine lessons. This spiral approach has two components. First, after every lesson, students reflect on their own thinking and summarize their thinking by writing down key ideas. Students are also asked to write down key ideas and thoughts that they themselves consider to be important to solve that day’s modelling task (we refer to this as TP3-1). Secondly, at the end of teaching program, the teacher made three groupings of the various modelling tasks that had been considered in the series of lessons. Then students were asked to identify common issues and approaches in each group of problems that assisted in the modelling process (we refer to this as TP3-2).

The first teaching principle (TP1) has already been emphasized in our previous study (Ikeda, Stephens and Matsuzaki, 2007), however (TP2) and (TP3) are newly introduced in this study.

2.2 Plan of a series of experimental teaching

The experimental teaching program emphasized the above three teaching principles, introduces a range of modelling tasks, and focused on key ideas that promote modelling. The following outlines nine teaching sessions starting in September and finishing in November of 2006:

1st lesson (September 2, Friday; 50min.):
- **principles:** (TP1) and (TP3-1) are emphasized.
- **task:** Three word problems (Whether or not can word problems be solved with proportional thinking?)
- **aim:** Understanding the necessity to link real problem situations to given word problems

2nd lesson (September 9, Friday; 50min.):
- **principles:** (TP1) and (TP3-1) are emphasized.
- **task:** Cake problem (How can you divide cakes in your family?)
- **aim:** Acquiring the idea of “Setting assumptions”

3rd lesson (September 9, Friday; 50min.):
- **principle:** (TP3-1) is emphasized.
- **task:** Cherry blossoms problem (How to predict when cherry blossoms bloom?)
- **aim:** Understanding a broad modelling process

4th lesson (September 15, Friday; 50min.):
- **principles:** (TP1) and (TP3-1) are emphasized.
- **task:** Checking counter problem (Should express checkout be introduced for customers who have purchased fewer than a certain number of items?) (Haines et al., 2001)
- **aim:** Acquiring the idea of “Clarifying the nature of problem”

5th lesson (September 15, Friday; 50min.):
- **principles:** (TP1) and (TP3-1) are emphasized.
- **task:** Super Market problem (What kinds of factors should be considered to estimate the number of check counters?) (Treilibs et al., 1980)
- **aim:** Acquiring the idea of “Generating and selecting variables”
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(6th lesson) (October 6, Friday; 100min.):
  **principles**: (TP1) and (TP2) and (TP3-1) are emphasized.
  **task**: Mirror problem (What size of mirror do I need at least in order to see my whole face?)
  (Shimada, 1990; Matsumoto, 2000; Ikeda, 2004)
  **aim**: Acquiring the idea of “Building up a mathematical model as simple as possible, and after building up the mathematical model modifying it more realistic gradually (Moscardini, 1989).”

(7th lesson) (October 20, Friday; 100min.):
  **principles**: (TP1) and (TP2) and (TP3-1) are emphasized.
  **task**: Age of dogs/cats (Let’s make formulas so that we can calculate the age of human beings that might correspond to the age of dogs/cats.)
  **aim**: Acquiring the idea of “Generating and selecting relations”

(8th lesson) (November 10, Friday; 50min.):
  **principles**: (TP1) and (TP2) and (TP3-1) are emphasized.
  **task**: Height of school (Let’s examine a student’s solution that calculated the height of school by applying the idea of similarity from the clipped picture taking a person standing in front of the school.)
  **aim**: Acquiring the idea of “Eliminating errors (Dealing with non-appropriate implicit assumptions)” and “Justifying the assumptions”

(9th lesson) (November 10, Friday; 50min.):
  **principle**: (TP3-2) is emphasized.
  **task**: Previous seven tasks are divided into three categories.
  What kinds of ideas are important in each category?
  **aim**: Reflecting the previous seven tasks and identifying the important ideas that promote modelling.

2.3 Assessment modelling task and related coding criteria

Before the series of experimental teaching, students considered the Heartbeat problem (PISA, 2006; pp. 74-76) which is as follows:

For Health reasons people should limit their efforts, for instance during sports, in order not to exceed a certain heartbeat frequency.

For years, the relationship between a person’s **recommended maximum heart rate** and the person’s **age** was described by the following **old formula**.

\[
\text{Old recommended maximum heart rate} = 220 - \text{age}
\]

Recent research showed that this formula should be modified slightly.

The **new formula** is as follows:

\[
\text{New recommended maximum heart rate} = 208 - (0.7 \times \text{age})
\]

Then, we let students answer the following specific questions devised by us and based on the Heartbeat problem and also one general question. These same questions were posed for students both before and after the series of experimental teaching.

**SQ1** (Specific question 1):
  New and old formulas were built up in “Heartbeat” problem. What are advantages do you think to build up these formulas?

**SQ2** (Specific question 2) [Interpretation]
  What kinds of points are suggested about physical training for people by being modified from old recommended formula to new recommended formula?

**SQ3** (Specific question 3) [Setting assumptions]
  What kinds of assumptions are set in this new recommended formula?

**SQ4** (Specific question 4) [Validating and modifying the assumptions]
When we use new recommended formula, what kinds of points do we need to pay attention?
If you might modify new recommended formula further, what kinds of points can you modify?

**GQ** (General question) [Which PROCESS is required?] [What makes a GOOD model?]
What kind of process is required to build up this kind of functional model?
What ideas are important when building up this kind of functional model?

Regarding the assessment of Specific questions and the General question, a graduated coding/scoring scheme was applied. For each specific question a four-point coding scheme was applied. Scoring of the General question will be discussed later.

**Specific question 1:** highest code is 3
- code 0: not understanding the meaning of problem
- code 1: to know the value that we want to get concretely. However, in this stage, real context is never addressed.
- code 2: to find out the maximum heartbeat value in a real world context. However, in this stage, the reason why it is advantage to know the maximum heartbeat value is never addressed.
- code 3: to find out the adequate momentum to keep one’s health, or to be able to do more effective exercise etc.

**Specific question 2:** highest code is 3
- code 0: not understanding the meaning of problem, no answer
- code 1: to be able to write the good points in general, but not to be able to write the variance of maximum heartbeat value according to the age, e.g. to know the maximum heartbeat value in detail, to be able to advice in detail, to be able to do sports more effectively.
- code 2: to be able to write the variance of maximum heartbeat value according to the age, namely, maximum heartbeat value of young people is get lower and one of old people is get higher. However, not to be able to write how people keep their health.
- code 3: to be able to write how people keep their health, namely, it suggests that young people do less practice, and that older people can exert themselves a little more.

**Specific question 3:** highest code is 3
- code 0: not to be able to write the following (x).
- code 1: to be able to write the following (x).
- code 2: to be able to write the following (a) or (b).
- code 3: to be able to write the following both (a) and (b).

  (x): the function is set approximately, or the function is set as average value
  (a): Writing about variables or data collections the maximum heartbeat value is only affected by age, and the function is set by using collected data.
  (b): Writing about relation between age and maximum heartbeat value is a straight line that decreases with age.

**Specific question 4:** highest code is 3
- code 0: not to be able to write the following (x).
- code 1: to be able to write the following (x).
- code 2: to be able to write the following (a) or (b).
- code 3: to be able to write the following both (a) and (b).

  (x): to do experiments repeatedly and modify the function. There is no concrete point to be modified.
  (a): Writing about variables or data collections to modify the variables by paying attention to sexual distinction, difference among individual, and so on in addition to the age, to increase the number of subjects, and so on.
  (b): Writing about relations to change the straight line into a smooth curve, to change slope and y-intersection of the function, to set up the range in the function, and so on.

**General question:** maximum score is 3

The following nine key ideas that promote modelling are set in this question. If students can write more than five items, then the score is 3. Similarly, writing three or four items is scored as 2, writing one or two items is scored as 1 and writing nothing is scored as 0.

(g1) Clarifying the nature of problem
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(g2) Generating and selecting variables
(g3) Collecting data
(g4) Generating and selecting relations
(g5) Interpreting and validating the model
(g6) Modifying the model repeatedly, Eliminating errors (Clarifying non-appropriate implicit assumptions)
(g7) Setting up assumptions, Clarifying ambiguous points
(g8) Justifying assumptions according to the aim of problem
(g9) building up a mathematical model as simple as possible, and after building up the mathematical model modifying it more realistic gradually.

For example in SQ3, a student’s writing “The maximum heartbeat value decrease 0.7 constantly in each year” is coded 2, because this writing only referred the relation between age and maximum heartbeat addressed in SQ3(b). On the other hand, a student’s writing “The maximum heartbeat value is only affected by age and the relation between age and maximum heartbeat value is not a curve but a straight line.” is coded 3, because this writing referred both the variables affecting maximum heartbeat value and the relation between age and maximum heartbeat value.

3. Implementation of the study in Kamakura Junior High School

Subjects were 34 9th grade students of Kamakura Junior high school who had selected mathematics as an elective subject. At the beginning of the series of lessons, the objectives were shown to students, namely to understand what kinds of ideas are important to solve a real world problem mathematically. After each teaching session, students were invited to write what kinds of ideas they thought to be important. This was intended to help students to reflect on their solving process, and to elicit key ideas that promote modelling. Three selected lessons are described below that focused on “Conflicting situations” (TP1), “Repeated connections” (TP2) and “Spiral reflections” (TP3).

3.1 Outline of 2nd lesson that focused on conflicting situation

The aim of the 2nd lesson on September 9 was to acquire the idea of “Setting assumptions”. The task used is as follows: There are five cakes. These cakes are divided in our family, namely grandfather, grandmother, father, mother, sister and me. How can I divide cakes in our family? After presenting this task, the teacher let students solve it by themselves, then to discuss their solution in a small group composed of 5 or 6 members. A variety of ideas were derived from students as follows.
(1) each person: 5/6, (2) sister and me: 3/2, other 4 persons: 1/2, (3) father: 3/2, sister and me: 1, grandfather, grandmother and mother: 1/2, (4) sister and me: 1, others: 1, (5) each person gets 1 by buying one more cake, (6) each person gets 1/2 by giving 2 cakes to neighborhood.

By realizing there are a variety of possible solutions, students pointed out that there are too few assumptions in this problem. However, they could not find out what to do next. This is the conflicting situation for students. Therefore, teacher divided six answers into three categories, namely first group is (1) only, second group is composed of (2), (3) and (4), and third group is composed of (5) and (6), then asked two questions such as “What kinds of assumptions do you need to set up so that everyone can get the answer of first group only?” , “What kinds of assumptions do you need to set up so that everyone can get the answers of first group or second group?” Students could find out the following assumptions about first question. Namely, “each cake is same size”, “the cakes are divided equally for each person” and “all of cakes are divided among six persons”. Regarding second question, they found out to need to set up two assumptions such as “the cakes are divided equally for each person” and “all of the cakes are divided among six persons”. Further, by focusing on how to cut the cakes, the necessity to set up the assumption that each person can get the same size and same shape of cake is discussed, teacher asked how to cut the cakes so that each person can get a chunk of cake. The answer “1/2 + 1/3” was derived from students. Students could realize the necessity to set up assumptions to get a unique solution, and consider what kinds of assumptions are required. By deriving the idea of “Setting assumptions” from the conflicting problem situation, it was intended for teacher to foster students’ ability to ask by themselves questions such as “What assumptions are used
in this problem?” “What assumptions do we need to solve this problem?”

3.2 Outline of 6th lesson that focused on repeated connection and conflicting situation

The aim of this 6th lesson on October 6 was to acquire the idea of “building up a mathematical model as simply as possible, and after building up the mathematical model modifying it more gradually to be more realistic.” In this lesson, the idea of “Setting assumptions” is focused on again. The task asks: Is the following sentence true or false? “Half size of mirror is needed at least in order to see my whole face”. After presenting the problem, the teacher let students predict an answer. One student said that it might be true because it seems to be half by drawing a figure. Another student said that it might be false because if the mirror is far from my face, it is sufficient to use small mirror. The teacher directed students to draw a figure to check their answer. The teacher asked Student A to draw a figure on the blackboard. Figure 1 was the resulting drawing. (The symbols A, B, C, D and E were added by the teacher.) Making students draw a figure helped them to identify further points that may have been unclear to them.

In fact, this drawing, shown in Figure 1, drew several opinions from other students such as “(1) How about the width of the face?”, “(2) Are three points, namely the point of the eye, the point of head and the point of chin, on a same line? “, “(3) Whether relation of two line, namely face and mirror, is parallel or not?”, “(4) Is the eye located at the midpoint between the point of head and the point of chin?”. The teacher commended students’ for these comments and questions because they highlight the necessity to clarify vague points in the problem and set up assumptions in order to get a unique solution just as last time. This means repeated connection.

For example, regarding the opinion (3), two conflicting opinions were derived from students. Some students said: “It seems to be easy to solve the problem if the relation of two lines is parallel”, “If the angle is not right angle, it is too difficult to solve the problem.” However, another student said: “However, the relation of two lines is not always parallel in a real situation”. This outcome is to be expected in creating conflicting situations. By considering this conflicting situation, the teacher was able to guide students’ opinions toward the following idea: “Let’s set up an assumption that the relation of two lines is parallel at first. Regarding the case of not parallel, let’s consider that later.” This idea is an instance of “building up a mathematical model as simply as possible, and after building up the mathematical model modifying it gradually to become more realistic”. Through this activity, students could understand that this idea is derived from the conflicting problem situation.

3.3 Outline of the last lesson that focused on spiral reflection

At the end of every lesson, students wrote what kinds of ideas are thought to be important to solve a real world problem. In this last lesson, students were asked to summarize their ideas that help promote modelling by reflecting back on their remarks written after each lesson. The teacher presented the following three questions by drawing attention to seven problems used in the series of lessons. This activity involves a spiral reflection that is intended to help students to summarize key ideas that promote modelling.

[First question] What kinds of ideas are thought to be important to translate a real world problem into a mathematical problem by reflecting the following problems?

(1) Cake problem (How can you divide cakes in your family?)
(2) Checking counter problem (Should express checkouts be introduced for customers who have purchased fewer than a certain number of items?)
(3) Mirror problem (What size of mirror do I need at least in order to see my whole face?)

[Second question] What kinds of ideas are thought to be important to interpret a mathematical solution in a real world by reflecting the following problems?

(1) Three word problems (Whether or not word problems can be solved with proportional thinking?)
(2) Age of dog/cat (Let’s make formulas so that we can calculate the age of human beings that might correspond to the age of dogs/cats.)
(3) Height of school (Let’s examine a student’s solution that calculated the height of school by applying the idea of similarity from the clipped picture taking a person standing in front of the school.)

[Third question] What kinds of ideas are thought to be important to build up a functional model by reflecting the following four problems?
(1) Super Market problem (What kinds of factors should be considered to estimate the number of check counters?)
(2) Cherry blossoms problem (When do cherry blossoms bloom out?)
(3) Three words problems (Whether or not can word problems be solved with proportional thinking?)
(4) Age of dog/cat (Let’s make formulas so that we can calculate the age of human beings that might correspond to the age of dogs/cats.)

At first, students wrote their own answers to the three questions, and then discussed their answers in small groups composed of four or five members. Finally, a representative of each small group presented a summary of answers and students shared important ideas.

4 Analysis of pre-test and post-test performances

4.1 Quantitative analysis about results of pre-test and post-test

Pre-test and post-test were done before and after a series of teaching. Two students were absent in pre-test and two students are absent in post-test. Therefore, there were 30 students who did both pre-test and post-test. The results of pre-test and post-test are as follows. Table 1 shows the number of students who were coded according to each criterion for each SQ (Specific question) 1-4 and GQ (General Question). In Table 1, E denotes the expected value. Contents of the four Specific questions and of the General Question were given in section 2(3) above.

<table>
<thead>
<tr>
<th>code</th>
<th>SQ1 code</th>
<th>SQ2 code</th>
<th>SQ3 code</th>
<th>SQ4 code</th>
<th>GQ code</th>
</tr>
</thead>
<tbody>
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<td>post</td>
<td>E</td>
<td>pre</td>
<td>post</td>
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<td>6</td>
<td>1</td>
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<td>25</td>
<td>20.5</td>
<td>1</td>
<td>6</td>
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</table>

Table 1: The result of pre-test and post-test.

A Chi-squared test was applied. The values of the Chi-squared test for SQ1-4 and GQ are shown in Table 2. The values are rounded off to one decimal place.

<table>
<thead>
<tr>
<th>Value of Chi-square</th>
<th>SQ1</th>
<th>SQ2</th>
<th>SQ3</th>
<th>SQ4</th>
<th>GQ</th>
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</thead>
<tbody>
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<td>11.0</td>
<td>7.5</td>
<td>37.9</td>
<td>32.2</td>
<td>50.1</td>
</tr>
</tbody>
</table>

Table 2: Results of Chi-squared test between pre-test and post-test.

In Table 1, for SQ1 and SQ2, the expected values of some cells are less than 5. Therefore it is not possible to do a Chi-squared test. Nevertheless, there are obvious improvements in students’ performances on SQ1 between the pre-test and post-test. These gains are not so clear in the case of SQ2. However, for SQ3, SQ4 and GQ, there are significant differences (df = 3, p<0.01), as a value of 11.3 on the Chi-squared test matches a probability of 0.01. From this analysis, it can be seen that the ideas of setting assumptions (SQ3) and validating and modifying the assumptions(SQ4) in a particular modelling problem (the Heartbeat problem) are significantly improved even though this modelling problem was not treated in the series of experimental teaching. Further, the ideas about general questions(GQ) such as “Which PROCESS is required?” and “What makes a GOOD model?” are also significantly improved through the series of experimental teaching. The experimental teaching based
on three teaching principles appeared to be very effective to foster students’ thinking that promotes modelling.

4.2 Qualitative Analysis about students’ writing

How can we see students’ progress qualitatively between pre-test and post-test? We illustrate this by samples of three students’ writing that exemplify meaningful progress in SQ4. Other students’ writing could be used to make the same point, and other convincing samples could be drawn from responses to Pre-test and Post-test writing for other Specific questions.

**Student K**

[pre-test]

I think that it is impossible for human being to represent accurate values about 208(y-intersection) and 0.7(slope). Therefore, it is recommended to find out more detail accurate values, just like 208.123 - {0.71×(age - 2)}.

[post-test]

- It is necessary to find out more detail accurate value about slope.
- If individual differences exist, it is necessary to take account of individual differences. For example, it is one of the methods to multiply an invariable by age by examining the breathing capacity.

Before the teaching program, this student only considered the accurateness of y-intersection and slope of linear function. However, after the teaching, this student considered the possibility to add one more variable, namely breathing capacity, and to take account of individual differences.

**Student M**

[pre-test]

- It is possible to modify 208(y-intersection) and 0.7(slope) of linear function.

[post-test]

- It is possible to modify 208(y-intersection) and 0.7(slope) of linear function.
- It might be necessary to modify straight line into curve.

Before teaching, this student only considered to modify y-intersection and slope of linear function. However, after the teaching, this student recognized the necessity to select the kind of relation fitting the real data, namely whether it might be a linear function or a curve.

**Student S**

[pre-test]

- The value 0.7 in “age×0.7” might be changed.

[post-test]

- The value 0.7 and 208 in the formula ”age×0.7+208” will be changed.
- Weight and height might be inserted in terms of age in the formula.

Before teaching, this student only wrote to modify the coefficient of linear function. However, after the teaching, this student saw the necessity to think about other variables that might influence the problem situation, namely weight and height.

Next, we focused on students’ responses to the General Question. The results of number of students who wrote each of nine key ideas that promote modelling is as follows (Contents of these nine key ideas are outlined in section 2(3) of the paper)

<table>
<thead>
<tr>
<th></th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
<th>G5</th>
<th>G6</th>
<th>G7</th>
<th>G8</th>
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<tr>
<td>Number of Students</td>
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<td>20</td>
<td>24</td>
<td>17</td>
<td>14</td>
<td>15</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>Percentage of students (%)</td>
<td>36.7</td>
<td>50.0</td>
<td>66.7</td>
<td>80.0</td>
<td>56.7</td>
<td>46.7</td>
<td>50.0</td>
<td>33.3</td>
<td>10.0</td>
</tr>
</tbody>
</table>

**Table 3**: Number of students who wrote about each of the nine key ideas

From table 3, items that more than 50% students could write are as follows; G4 [80.0%]
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(Generating and selecting relations), G3 [66.7%] (Collecting data), G5 [56.7%] (Interpreting and validating the model), G2 [50.0%] (Generating and Selecting variables) and G7 [50.0%] (Setting up assumptions, Clarifying ambiguous points). As more than 50% students could write these five items in their free writing, this experimental teaching was said to be effective for students in these five items. However, G1 [36.7%] (Clarifying the nature of problem), G8 [33.3%] (Justifying assumptions according to the aim of problem) and G9 [10.0%] (Building up a mathematical model as simple as possible, and after building up the mathematical model modifying it more realistic gradually) are said to be difficult for students.

Next we select three students who made a meaningful progress in General Question between the pre-test and the post-test.

**Student I**

[pre-test]
- Wrote nothing.

[post-test]
- Translating a real world problem into mathematical problem. In this stage, it is important to set up simple clear assumptions. Vague things should be deleted.
- Solving problem mathematically. In this stage, we solve the problem mathematically, not relating to the real world situation, based on the assumptions. Further, we need to consider the problem according to the aim not to leap the logic.
- Representing the real situation as a mathematical function. In this stage, it is OK to represent the function approximately according to the aim, because some situations are too complex. It is important to consider the degree how we can allow the error according to the aim.

Before teaching, this student wrote nothing. However, after the teaching program, this student described three processes involving important key ideas of modelling, namely, setting up assumptions by eliminating vague things, building up a mathematical model as simple as possible, clarifying the range how we can allow the error according to the aim of original problem.

**Student K**

[pre-test]
- Wrote nothing.

[post-test]
- Clarifying what kind of solution we want to find out before building up a function.
- Considering what kind of data, conditions and so on are necessary to build up function.
- Setting up assumptions and formulating a formula after correcting the data, conditions and so on.
- After building up the formula, validating the formula with a real corrected data.
- If there is a mismatch, modifying the formula. (changing assumptions or reformulating the formula)
- Repeating 4 and 5.
  (Even through there exist some mismatches between the formula and a real corrected data, it is important to allow them and minimize the mismatches as less as possible between the formula and a real corrected data.)

Before teaching, this student wrote nothing. However, after the teaching program, this student described six processes involving important key ideas of modelling, namely, clarifying the nature of problem, setting up assumptions, validating assumptions according to the aim of problem, modifying the model and repeating the validating and modifying process until getting an adequate solution.

**Student A**

[pre-test]
- It is necessary to collect a variety of data and take the average. We should take account of individual differences.

[post-test]
- Setting up assumptions by following the next three steps.
- Finding up the variables influencing the problem situation.
− Considering whether the relation is straight line or curve.
− Considering how we can set up the coefficient of the function.

At first, we set up the assumptions about (1), then correct data and determine about (2). In this stage, it is important to determine whether we select straight line even though there is a mismatch between the function and the data or we select curve so that the function can represent the detail differences of the real data according to the aim.

Before teaching, this student only considered how to collect data in order to take account of individual differences. However, after the teaching program, this student was able to refer to the importance of considering how to generate and select variables, how to generate and select relations and how to set coefficient of the function. Further, when generating and selecting relations, this student said it was important to choose whether we select a straight line to prefer the simplicity or we select a curve to prefer the accuracy according to the aim of original problem.

5 Conclusion

This study reports on an experimental teaching program which consisted of nine lessons for Japanese 9th grade students. In this program, the following three teaching principles were emphasized; (TP1) Conflicting Situations, (TP2) Repeated Connections and (TP3) Spiral Reflections.

Using the PISA (2006) Heartbeat problem as a pre- and post-test question, and a general question on modelling we examined what transfigurations or shifts in students’ knowledge of modelling could be observed as a result of the experimental teaching program. This general question had the following sub-questions: What processes are required to build up a functional model? What ideas are important for developing an effective functional model? A graduated coding analysis was applied to assess students’ responses to the Specific (Heartbeat) question and to the General question.

Significant gains in students’ post-test performances on Specific and General questions were obtained. This and the resulting discussion provide strong evidence that a teaching program based on the above three teaching principles is substantially effective in fostering students’ thinking to promote modelling. This conclusion is powerfully reinforced when one examines written examples of students’ thinking, produced in the study, which consistently demonstrate marked increases in the quality and depth of their insight into the modelling process.

References


Three teaching principles for fostering students’ thinking about modelling:
An experimental teaching program for 9th grade students in Japan

